THERMAL INSULATION OF BUILDINGS IN A HOT DRY CLIMATE WITH SPECIAL REFERENCE TO THE SUDAN.

SID AHMED ABDALLA IBRAHIM

Ph.D. THESIS

UNIVERSITY OF EDINBURGH

SEPTEMBER 1968

ACKNOWLEDGEMENTS

The author wishes to acknowledge indebtedness, and express gratitude, to the supervisors of his studies: Professor Sir Robert Mathew, former professor of Architecture, University of Edinburgh, and Professor O. W. Hendry, professor of Civil Engineering, University of Edinburgh.

The author has been given invaluable help and guidance by Professor C. B. Wilson, first at the Building Climatology Research Unit of the department of Building Science, University of Liverpool, and later as professor of Architectural Science at the department of Architecture, University of Edinburgh, for which grateful thanks are expressed.

Thanks are also due to Mr. P. F. Crofts, advisor to Post-Graduate students at the department of Architecture, University of Edinburgh, for advice and guidance given throughout the period of studies.

While at the department of Building Science of the University of Liverpool, the author received valuable assistance, on some analytical aspects of the studies, from Dr. M. G. Davies. Mr. K. Thompson, formerly of that department, helped the author to an understanding of the Analogue Simulator on which the experimental programme connected with this work was carried out. This assistance is gratefully acknowledged.

The author's thanks are also due to Mr. Bill Baker, and Mr. Jim MacMillan, technical staff at the department of Building Science, University of Liverpool, for help rendered with the experimental work, and to Mr. Arthur Morris, also of the technical staff of that department, for preparing photographic reproductions of the illustrations in this text.

SUMMARY

An Analogue Simulator is used to study the problems of natural heating and thermal insulation of buildings in a hot dry climate.

A discussion is given on the way different climatic factors enter into the heat exchange process. The Analogue Simulator set-up is described and the theory of thermal circuitry and analogue simulation outlined.

Weather design data selected for Khartoum, Sudan, are presented. The thermal comfort of people in a hot dry climate is briefly discussed.

Building elements of different types of construction, and an enclosure formed of selected wall and roof sections, on which the experimental programme is based, are described and the thermal circuits given.

The temperatures of uninsulated building elements are predicted under different conditions of exposure, for both conditioned and unconditioned buildings. Insulation is applied in various amounts, and in different positions, to wall and roof elements, and a prediction of temperatures obtained at each step.

It is concluded that although, in the hot dry climate considered, thermal comfort conditions cannot be maintained in buildings without mechanical cooling, heavy structures are capable of a better thermal performance than lighter ones.

In unconditioned buildings, while insulation placed on the weather side of a building element leads to a reduction in peak temperatures and temperature fluctuations at the inside surface, the effect of insulation placed on the inside surface depends on the heat capacity of the structure and the magnitude of internal heat gain. In this case insulation is more useful when applied to elements acting as heat donors, rather than heat sinks.

In conditioned buildings, where the indoor air temperature is kept constant, insulation, whether placed on the outside or inside surfaces of building elements, has the same effect on, and tends to reduce the mean of, the inside surface temperatures.

LIST OF CONTENTS

INTRODUCTION

To design for optimum environment in buildings is probably no easier in a cold climate than it is in a warm one. Failure to achieve the desired environment will, however, probably result in more hardship in a warm climate. More latitude exists in a 'too cool' environment for personal adjustment of clothing or posture. When a warm environment becomes 'too warm', all one can do is to shed one's clothing: one does not usually have much to shed.

More is required of the 'tropical' designer than his 'temperate' counterpart.

This is probably one reason why study of the thermal behaviour of buildings in warm climates and the search for ways of living with heat (without dying of it) has a long way to go. As it is today, much less is known about designing for heat than is known about designing for cold.

Since this author will probably be obliged, for the rest of his life, to accept the too generous proffering of the desert sun, he has tried to do his bit in this work on the thermal insulation of buildings in hot dry climates.

SYMBOLS

T, T_1, T_2 = temperatures, ^{o}F.

T_{sa} = sol-air temperature, ^{o}F.

T_e = mean radiant environmental temperature, ^{o}F.

T_a = air temperature, ^{o}F.

T_o = outdoor air temperature, ^{o}F.

T_i = indoor air temperature, ^{o}F.

T_{so} = outside surface temperature, ^{o}F.

T_{si} = inside surface temperature, ^{o}F.

τ = absolute temperature, ^{o}R, suffixes as above.

h = coefficient of heat transfer at a surface, Btu/ft.^{2} hr.^{o}F.

h_o = coefficient of heat transfer at an outside surface of a building element, Btu/ft.^{2} hr.^{o}F.

h_i = coefficient of heat transfer at an inside surface of a building element, Btu/ft.^{2} hr.^{o}F.

h_c = coefficient of heat transfer by convection at a surface, Btu/ft.^{2} hr. ^{o}F.

h_{co} = coefficient of heat transfer by convection at an outside surface of a building element, Btu/ft.^{2} hr. ^{o}F.

h_{ci} = coefficient of heat transfer by convection at an inside surface of a building element, Btu/ft.^{2} hr. ^{o}F.

h_r = coefficient of heat transfer by long wave radiation at a surface, Btu/ft.^{2} hr. ^{o}F.

h_{ro} = coefficient of heat transfer by long wave radiation at an outside surface of a building element, Btu/ft.^{2} hr ^{o}F.

h_{ri} = coefficient of heat transfer by long wave radiation at an inside surface of a building element, Btu/ft.^{2} hr.^{o}F.

R = resistance to heat transfer at a surface = $\frac{1}{h}$, ^{o}F.hr.ft.^{2}/Btu, suffixes as above.

k = thermal conductivity, Btu in./ft.^{2} hr.^{o}F.

ρ = density, lb/ft.^{3}

c = specific heat, Btu/lb. ^{o}F.

a = thermal diffusivity, ft.^{2}/hr.

l = thickness of building element, in the direction of heat flow, inch.

A = area, ft.^{2}

V = volume, ft.^{3}

t = time, hr.

I = intensity of, short wave, solar radiation falling on a surface, Btu/ft.^{2} hr.

I_D = intensity of the direct component of solar radiation, Btu/ft.^{2} hr.

I_d = intensity of the diffuse component of solar radiation, Btu/ft.^{2}hr.

I_{DH}, I_{DV} = intensity of the direct component of solar radiation falling on a horizontal and a vertical surface, respectively, Btu/ft.^{2} hr.

I_{dH}, I_{dV} = intensity of the diffuse component of solar radiation falling on a horizontal and a vertical surface, respectively, Btu/ft.^{2} hr.

I_N = intensity of, short wave, solar radiation, normal to the sun's rays, Btu/ft.^{2} hr.

I_g = intensity of short wave, solar radiation reflected by ground Btu/ft.^{2} hr.

α = absorptivity of surface to, short wave, solar radiation, dimensionless.

r = reflectance of ground to, short wave, solar radiation, dimensionless.

β = solar altitude angle, degree.

γ = surface - solar azimuth angle, degree.

θ = angle of incidence, degree.

I_{lw} = long wave radiation emitted, or received by a surface, Btu/ft.^{2}hr.

ϵ = emissivity of a surface to long wave radiation, dimensionless.

σ = Stephan-Boltzmann constant = 0.173×10^{-8}, Btu/ft.^{2}hr.$^\circ R^4$.

R_{sky} = long wave radiation emitted by atmosphere, falling on a surface, Btu/ft.^{2}hr.

R_{gro} = long wave radiation emitted by ground, falling on a surface, Btu/ft.^{2} hr.

p = vapour pressure at ground level, millibar.

λ = wave length, ft.

f = frequency, cycles/hr.

R_e = electric resistance, ohm.

E = electric potential, volt.

V = potential difference, volt.

i = electric current, ampere.

Q_e = electric charge, coulomb.

C_e = electric capacitance, farad.

R_t = thermal resistance, °F. hr.ft.^{2}/Btu.

q = heat flow, Btu/hr.

w = heat flux, Btu/ft.^{2}hr.

Q_t = thermal energy, Btu.

C_t = thermal capacity, Btu/°F.

R_v = resistance to ventilation heat flux, °F.hr/Btu.

F_e = emissivity factor, dimensionless.

F_{1-2} = shape factor, dimensionless.

CHAPTER 1

HEAT TRANSFER AND THERMAL INSULATION

1.1. HEAT TRANSFER

1.1.1. General

A building and the environment in which it sits form a complex thermal system. The outdoor environment acts on the building setting up convective, conductive, and radiative heat exchange processes. The building acts as a filter modifying the effect of the outdoor environment on the indoor climate.

The weather-side of a building element may absorb short wave solar radiation, emit to, and receive from the atmosphere ground and other objects, long wave radiation, exchange heat by convection and conduction with air, and gain or lose heat by the absorption or evaporation of moisture. Heat is conducted through the building material and exchanged at the inside surface of an element by convection and conduction, with the inside air and by long wave radiation with other surfaces and objects.

Since the weather factors involved in these heat exchange processes vary with time in an irregular manner, the heat exchange processes between the building and the environment are transient in nature and the resulting temperature distribution in the building fabric is a complicated function of time, never repeating itself exactly.

The exact analysis of such thermal systems is a very involved matter, and it is customary to make certain assumptions regarding the heat exchange processes so as to simplify the analysis.

1.1.2. Steady state heat flow

The assumption is sometimes made that the average outdoor and indoor air temperatures stay constant for the 24-hour period of the day. The heat flow calculations are then made for a constant temperature difference,[1] with certain factors to correct for solar radiation heat gains. In this case the heat flow calculations are relatively simple and the flow is assumed to be steady so that the temperature distribution in the building element does not very with time and there is no change in the heat content of the element. The heat that enters one side of the element, leaves through an equal area of the other side. The equation giving the heat flow through a building element under these conditions derives from the fundamental assumption of heat conduction theory, which is that if heat is flowing in the direction of the x-axis, the temperature being the same over any plane x and constant, and if (T) be the temperature at point x, then the heat (w) flowing per unit time, per unit area across the plane x is:[2]

$$w = -k\frac{\partial T}{\partial x} \quad \ldots\ldots \quad (1.1)$$

where k, the thermal conductivity, is a constant for the medium which is numerically equal to the quantity of heat that flows in unit time through unit area of the element of unit thickness for unit temperature difference across the thickness

$\frac{\partial T}{\partial x}$ is the temperature gradient at the plane. The negative sense in equation (1.1) is due to the fact that the temperature gradient taken in the direction of 'positive' heat flow is intrinsically negative.

---

1. Billington, N.S., "Thermal properties of buildings". Cleaver-Hume Press Ltd., 1952.

2. Jaegar, J.C., "An introduction to applied mathematics". Oxford University Press, 1963.

For an element of thickness (l), and constant surface temperatures T_1 and T_2 and conductivity (k), equation (1.1) gives the conduction equation in the steady state for the heat flow (w) in one direction per unit time per unit area as:

$$w = k\,\frac{T_1 = T_2}{l} \quad \ldots\ldots\ldots\ldots \quad (1.2)$$

A more useful equation is that which gives the heat flow in terms of the outside and inside air temperatures, since surface temperatures are not usually known. This is arrived at by equating the heat flows through the outside surface, across the element and through the inside surface. Since there is no storage of heat under conditions of steady state heat flow, all these flows must be equal so that:

$$w = h_o(T_o - T_1) = \frac{k}{l}(T_1 - T_2) = h_i(T_2 - T_i) \quad \ldots\ldots\ldots \quad (1.3)$$

where T_o, T_i = temperatures of the outdoor and indoor air, respectively.

h_o, h_i = surface conductances, at the outside and inside surfaces respectively.

Equation (1.3) can also be written as:

$$w = \frac{T_o - T_1}{R_o} = \frac{T_1 - T_2}{R} = \frac{T_2 - T_i}{R_i} \quad \ldots\ldots\ldots \quad (1.4)$$

Where R_o, R_i = surface resistances equal $\frac{1}{h_o}$ and $\frac{1}{h_i}$ respectively.

R = resistance of element equal to $\frac{l}{k}$.

From equation (1.4) it will be seen that

$$w = \frac{T_o - T_i}{R_o + R + R_i} = \frac{T_o - T_i}{R_t} \quad \ldots\ldots\ldots \quad (1.5)$$

where $R_t = R_o + R + R_i$ and is the total outdoor air to indoor air resistance to heat flow.

The reciprocal of the total resistance (R_t) is known as the U-value, or thermal transmittance,* denoted by (U), so that equation (1.5) can be written as:

$$w = U(T_o - T_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.6)$$

U has the dimension Btu/ft.^{2} hr.^{o}F.

To allow for the effect of solar radiation the sol-air temperature (T_{sa}) can be used instead of the outdoor air temperature (T_o) in equation (1.6) which becomes

$$w = U(T_{sa} - T_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.7)$$

The sol-air temperature was first introduced by Mackey and Wright[3] and defined as the hypothetical temperature of the outdoor air which in contact with a shaded building element, which does not directly transmit solar radiation, would give the same rate of heat flow and the same temperature distribution through the element as exists with the actual outdoor air temperature and incident solar radiation. It was expressed as

$$T_{sa} = T_o + \frac{\alpha I}{h_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.8)$$

where I = intensity of solar radiation in Btu/ft.^{2} hr.

α = absorptivity of surface to solar radiation, dimensionless.

h_o = the outside surface conductance.

Equation (1.8) has, however, since been adjusted to include the effect of long wave radiation exchange and was given by Roux[4] as:

$$T_{sa} = T_o + \frac{\alpha I - I_{lw}}{h_{co}} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.9)$$

---

* loc.cit. See footnote 1.1

3. Mackey, C.O. and Wright, L.T.Jr., "Summer comfort factors as influenced by the thermal properties of building materials". A.S.H.V.E. Transactions, Vol.49, 1943.

4. Roux, A.J.A. "Periodic heat flow through building components - Heat exchange at the outside surface with special reference to the application of sol-air temperature". N.B.R.I. Pretoria, S. Africa. 1950.

where

I_{lw} = the difference between long wave, low temperature, radiation emitted, and received, by the surface in Btu/ft.^{2} hr.

h_{co} = outside surface convection heat transfer coefficient in Btu/ft.^{2} hr.^{o}F.

and was given in another form by Page[5] as:

$$T_{sa} = T_o + \frac{\alpha I}{h_{co} + h_{ro}} + \frac{h_{ro}(T_e - T_o)}{h_{co} + h_{ro}} \quad \text{..........} \quad (1.10)$$

where T_e = mean radiant environmental temperature

h_{ro} = radiation transfer coefficient for the outside surface (calculated from Stephan's law for a surface of known emissivity).

h_{co} = outside surface convection heat transfer coefficient.

When steady state heat flow is assumed, two building elements will have identical heat flow characteristics if they have:

(a) the same ratio of thickness to thermal conductivity $(\frac{l}{k})$ as can be seen from equation (1.2)

(b) the same solar absorptivity and low temperature emissivity, as they will have the same sol-air temperatures, (equation (1.10)), and the same surface conductances, and hence the same U-value, (equation (1.5)).

However, because building materials of any finite thickness have a heat storage capacity, steady state conditions can only exist if*:

(a) The temperature and motion of outdoor and indoor air remains constant with respect to time.

(b) The net heat exchange by short wave and long wave radiation at the outside surface of an element remains constant with respect to time.

---

5. Page, J.K., "A technical note on the calculation of temperatures in concrete roof slabs". RILEM symposium on concrete and reinforced concrete in hot countries, Haifa, 1960.

* loc.cit. See footnote 1.3.

(c) The net heat exchange by long wave radiation at the inside surface of the element remains constant with respect to time.

These conditions seldom exist in practice over any length of time, and it has been shown by Houghton, Zobel, et al.[6,7,8] that large errors can be introduced in heat flow calculations by ignoring the unsteady nature of heat flow especially under conditions where large diurnal variations of solar radiation, air temperature and wind speed may occur.

1.1.3. Periodic heat flow.

While the rate at which heat is transferred in a body is dependent on the conductivity of the material, (equation (1.1)), there is a rise in temperature produced by this heat, which varies with the density and specific heat of the medium.

The differential equation governing the conduction of heat, under unsteady conditions, in a homogeneous body and giving the rate of temperature change in the material for three-dimensional heat flow, is known as the Fourier conduction equation which is written:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad \ldots\ldots\ldots\ldots (1.11)$$

For one-dimensional heat flow equation (1.11) reduces to:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.12)$$

---

6. Houghten, F.C. and Zobel, C.G., "Heat transfer through a roof under summer conditions". A.S.H.V.E. Transactions, Vol. 34, 1928.

7. Houghten, F.C., and Zobel, C.G., "Coefficients of heat transfer as measured under natural weather conditions". A.S.H.V.E. Transactions, Vol. 34, 1928.

8. Houghten, F.C., and Zobel, C.G., et al., "Additional coefficients of heat transfer under natural weather conditions". A.S.H.V.E. Transactions, Vol. 35, 1929.

where, k = conductivity of the material

ρ = density of the material

c = specific heat of the material.

the term $\frac{k}{\rho c}$ is known as the diffusivity of the material.

To find the temperature distribution in an element, equation (1.12) has to be solved with the appropriate initial and boundary conditions. It has been found that the solution of the equation is simpler if the boundary conditions are assumed to vary in a cyclic manner with time, being repeated every complete period (for in this case a boundary variable can be expressed in a Fourier series, as a constant plus a series of sinusoidal variations of different frequencies[9].) When this condition is assumed the heat flow is said to be periodic.

Under periodic heat flow the temperature at any point in the medium varies in a cyclic manner duplicating itself each complete period. This is, of course, also an idealisation which may not represent exactly what occurs in natural systems.

It has, however, been demonstrated by Houghten, Blackshaw, Pugh and McDermott[10], who investigated heat flow through several panels under unsteady conditions that good agreement can be obtained between experimental and analytical results, by solving the differential conduction equation for a known variation of the outside surface temperature and a constant inside air temperature.

---

9. See for example: Alford, J.S., Ryan, J.E., and Urban, F.O., "Effect of heat storage and variation in outdoor temperature and solar intensity on heat transfer through walls". A.S.H.V.E. Transactions, Vol. 45, 1939.

10. Houghten, F.C., Blackshaw, J.L., Pugh, E.M. and McDermott, P., "Heat transmission as influenced by heat capacity and solar radiation". A.S.H.V.E. Transactions, Vol. 38, 1932.

Using the data of Houghten et al., Alford, Ryan and Urban*, solved the equation for a known variation of the outdoor air temperature and solar radiation, both assumed to vary in a cyclic manner every 24-hour period, and a constant inside air temperature. A good agreement was reported between test and analytical results.

Mackey and Wright[**,11] developed the solution of Alford, Ryan and Urban for an expression of the heat exchange at the outside surface in terms of the sol-air temperature.

An extensive experimental programme, at the National Building Institute of S. Africa, undertaken by Roux et al.[***,12,13,14,15,16] to test the validity of the

---

* loc.cit. see footnote 1.9

** loc.cit. see footnote 1.3

*** loc.cit. see footnote 1.4

11. Mackey, C.O., and Wright, L.T. Jr., "Periodic heat flow - Homogeneous walls or roofs". A.S.H.V.E. Transactions, Vol. 50, 1944.

12. Roux, A.J.A., "A test room for the study of heat transmission through building components under conditions of periodic heat flow". N.B.R.I., Pretoria, S. Africa, 1949.

13. Roux, A.J.A., "Periodic heat flow through building components - Heat exchange at the inside surface of a wall panel". N.B.R.I. Pretoria, S. Africa. 1950.

14. Roux, A.J.A., "Periodic heat flow through building components - Heat transfer from the outside surface of homogeneous wall panels to the inside air". N.B.R.I. Pretoria, S. Africa. 1950.

15. Roux, A.J.A., "Periodic heat flow through building components - Heat transfer from the outside surface of homogeneous wall panels to the inside air under winter conditions". N.B.R.I., Pretoria, S. Africa. 1950.

16. Roux, A.J.A., et al. "Periodic heat flow through building components - heat transfer through homogeneous wall panels from the outdoor climate environment to the indoor air". N.B.R.I., Pretoria, S. Africa. 1951.

assumption of periodic heat flow under conditions of high solar radiation intensity and large diurnal temperature variations led to the conclusion that periodic heat flow theory in the forms proposed by Alford, Ryan and Urban, and Mackey and Wright, can be applied with a satisfactory degree of accuracy provided the correct expression for heat exchange at the outside surface (sol-air temperature as given by equation (1.9)) is used, and appropriate values allotted to the surface coefficients.

1.1.4. Heat exchange at the outside surface of a building element

At the outside surface of a building element heat may be transferred to or from the surface by any of the following ways.

(1) absorption of short wave radiation direct from the sun or diffuse from the sky, ground and other objects.

(2) long wave radiation exchange with the sky, ground and other objects.

(3) convection from the outdoor air.

(4) absorption, evaporation or internal redistribution of moisture.

(5) conduction through the building material.

These factors will now be dealt with in turn:

1.1.4.1. Solar radiation

An unshaded horizontal surface may receive short wave radiation direct from the sun and diffuse from the sky vault. The heat absorbed by the surface per unit time per unit area of the surface is:

$$\alpha_\theta I_D + \alpha' I_d$$

where:

α_θ = absorptivity of surface for direct solar radiation, at an angle of incidence θ, dimensionless.

α' = absorptivity of surface for diffuse sky radiation, dimensionless.

I_D, I_d = Intensity of direct and diffuse short wave radiation incident on the surface, respectively, in Btu/ft.^{2} hr.

A vertical or inclined surface may receive in addition, short wave radiation reflected from the ground. Denoting the ground reflected radiation falling on a surface by I_g and the absorptivity of the surface to this radiation by (   ), we have the heat (w_1) absorbed by any surface as:

$$w_1 = \alpha_\theta I_D + \alpha' I_d + \alpha'' I_g \quad \text{..........} \quad (1.13)$$

The solar absorptivity, which is a property of a particular surface, may, for some materials, be different for reflected and direct radiation due to the difference in the spectral composition of the radiation. Furthermore, the absorptivity for direct radiation varies with the angle of incidence of the radiation on the surface. Whereas it is slightly affected by changes in the angle between 0 and 60°, it drops markedly for any further increase in the angle.*

It has however been shown* that no serious error is introduced in heat flow calculations by assuming the absorptivity for all short wave radiation to be constant at the value for normal incidence ($\alpha_{\theta = 0}$). Denoting this value of absorptivity by α, equation (1.13) can be written:

$$w_1 = \alpha I \quad \text{..........................} \quad (1.14)$$

where

$$I = I_D + I_d + I_g$$

---

* loc.cit. See footnote 1.4.

1.1.4.2. Long wave radiation

All bodies continuously emit long wave radiation at a rate given by Stephan-Boltzmann law:

$$I_{lw} = \sigma \epsilon \tau_s^4 \quad \text{.....................} \quad (1.15)$$

where,

I_{lw} = long wave radiation emitted by surface, Btu/ft.^{2} hr.

ϵ = emissivity of surface for longwave radiation, defined for a specific temperature as the ratio of the emissive power of the surface at that temperature to the emissive power of a black body at the same temperature.

σ = Stephan-Boltzmann constant, equal to 0.173×10^{-8} Btu/ft.^{2} hr. $^{\circ}R^4$.

τ_s = the absolute temperature of the surface in $^{\circ}R = (^{\circ}F + 459)$.

A building surface also receives long wave radiation from the sky, ground, and neighbouring objects. The absorptivity of a surface to long wave radiation is, by Kirchoff's law, equal to the emissivity of the surface for radiation of the same wavelength. Although the incident and the emitted radiations may not have identical spectral distribution the absorptivity and emissivity of most building materials for the range of long wave radiation encountered at normal temperatures may be considered equal and constant at the spectral average value.[17]

The long wave radiation emitted by a clear atmosphere was correlated with absolute humidity by Brunt[18] who gave the empirical relationship:

$$R_{sky} = R_{bo} (a + b\sqrt{p}) \quad \text{.........} \quad (1.16)$$

17. Holden, T.S., "Calculation of incident low temperature radiation upon building surfaces." A.S.H.R.A.E. Journal, Vol. 3, No.4, 1961.

18. Brunt, D., "Notes on radiation in the atmosphere". Quarterly Journal, Royal Meteorological Society, Vol. 58, 1932.

where,

R_{sky} = long wave radiation emitted by the atmosphere, Btu/ft.^{2} hr.

R_{bo} = long wave radiation emitted by a black body at screen air temperature, Btu/ft.^{2} hr.

p = water vapour pressure at ground level.

a,b = constants which vary to some extent, their mean values being 0.55 and 0.056, respectively, when the vapour pressure is expressed in millibars.

It has, however, been argued by Swinbank,[19] that the apparent correlation found by Brunt between atmospheric radiation and vapour pressure could be attributed to the correlation between temperature and humidity at low levels, and that long wave radiation from clear skies is actually related only to the screen air temperature, the regression equation being:

$$R_{sky} = 5.31 \times 10^{-14}\ \tau_o^6 \quad (\text{millwatt/cm}^2) \ \dots\dots\dots\dots \ (1.17)$$

where,

τ_o = absolute temperature of air in ^oK.

The ground emits radiation at the rate (R_{gro}), given by equation (1.15) as:

$$R_{gro} = \epsilon_{gro}\sigma\tau_{gro}^4 \ \dots\dots\dots\dots \ (1.18)$$

where,

ϵ_{gro} = emissivity of ground to long wave radiation

τ_{gro} = absolute temperature of ground.

The rate at which a building surface receives radiation from the ground depends on the angle (ψ) between the surface and the ground and is given by:*

19. Swinbank, W.C., "Long-wave radiation from clear skies". C.S.I.R.O., Division of meteorological physics, Aspendale, Australia, 1963.

* loc.cit. See footnote 1.17.

$$R' = \frac{1}{2} R_{gro}(1 + \cos\psi) \quad .... \quad (1.19)$$

where,

R' = ground radiation falling on a particular surface making angle with the ground.

R_{gro} = ground radiation given by Equation (1.18).

For a horizontal surface ($\psi = 180^\circ$) equation (1.19) gives:

$$R' = 0$$

For a vertical surface ($\psi = 90^\circ$):

$$R' = \frac{1}{2} R_{gro} \quad .... \quad (1.20)$$

This agrees with the relationship given by Roux* which gives the radiation falling on a vertical surface from the sky and ground as:

$$R'' = F'R_{sky} + F'' R_{gro} \quad .... \quad (1.21)$$

where,

R'' = the total long wave radiation falling on a vertical surface, $Btu/ft.^2\, hr$, radiation from sky and ground treated as black body radiation.

F', F'' = angle factors with respect to the surface, of the sky and ground respectively.

In the absence of obstructions Roux gives

$$F' = F'' = 0.5.$$

The net long wave exchange for a horizontal surface can be written as:

$$w'_2 = \epsilon\,(\sigma T^4_{so} - R_{sky}) \quad .... \quad (1.22)$$

---

* loc.cit. See footnote 1.4.

where,

w'_2 = net heat exchange at a horizontal surface due to long wave radiation, Btu/ft.^{2} hr.

ϵ = emissivity of surface

τ_{so} = absolute temperature of surface, $^\circ$R.

R_{sky} = long wave radiation incident on surface from the sky.

and for a vertical surface from equation (1.21):

$$w''_2 = \epsilon \left(\sigma \tau_{so}^4 - \tfrac{1}{2}(R_{sky} + R_{gro}) \right) \text{ .... } (1.23)$$

where,

w''_2 = the net exchange by long wave radiation for a vertical surface, Btu/ft.^{2} hr.

ϵ = emissivity of the surface.

τ_{so} = the absolute surface temperature, $^\circ$R.

R_{sky}, R_{gro} = the long wave radiation emitted by the sky and ground, respectively, in Btu/ft.^{2} hr.

Alternatively, if the environment seen by the surface radiates at a temperature (T_e) the net exchange for any surface can be expressed by:[20]

$$w_2 = \epsilon \sigma (\tau_{so}^4 - \tau_e^4) \text{ .... } (1.24)$$

where,

w_2 = the net exchange by long wave radiation for any surface, Btu/ft.^{2} hr.

ϵ = emissivity of surface

τ_{so} = absolute temperature of surface, $^\circ$R.

τ_e = absolute radiant temperature of the environment 'seen' by the surface, $^\circ$R.

---

20. Hoglund, B.I., Mitalas, G.P., and Stephenson, D.G. "Surface temperatures and heat fluxes for flat roofs". Building Science, Pergamon Press, March, 1967.

Introducing a long wave radiation coefficient for an outside surface (h_{ro}), defined as the rate of heat transfer due to long wave radiation exchange per unit time, per unit area, for unit temperature difference between surface and environment, the net long wave exchange (w_2) can be written as:

$$w_2 = h_{ro}\,(T_{so} - T_e) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.25)$$

where,

h_{ro} = outside surface coefficient for long wave radiation, Btu/ft^2.hr.$^{\circ}$F.

T_{so}, T_e = surface temperature and the radiant temperature of the environment, respectively, in $^{\circ}$F.

It follows from equations (1.24) and (1.25) that:

$$h_{ro} = \frac{\epsilon\sigma\,(\tau_{so}^4 - \tau_e^4)}{T_{so} - T_e}$$

$$h_{ro} = \quad (\tau_{so} + \tau_e)\,(\tau_{so}^2 + \tau_e^2) \quad \ldots\ldots \quad (1.26)$$

For a horizontal surface the radiant environmental temperature will be equal to the radiant temperature of the sky, (see equation (1.22)). For a vertical surface, assuming the radiation coefficient (h_{ro}), (equation (1.26)), has the same value for radiation exchange with the sky and ground, from equation (1.23):

$$T_e = \frac{T_{sky} + T_{gro}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.27)$$

So that the environmental temperature for a vertical surface is the average of the sky and ground temperatures.

The expression for heat exchange, due to long wave radiation, given by equation (1.25) was used in this text, with h_{ro}, the radiation coefficient evaluated from equation (1.26).

1.1.4.3. Convection

Air in contact with the surface of a building element may lose heat to, or gain heat from, the surface by conduction. This process leads to density changes in the air which cause it to move and mix. This motion entails removal of heat from, or addition of heat to, the surface of the element.

The heat transfer due to the movement of air over a surface is called convection heat transfer and can be expressed as:

$$w = h_c (T_s - T_a) \quad \text{............} \quad (1.28)$$

where

w = net heat transfer by convection from the surface to the air per unit time per unit area of surface.

h_c = convection heat transfer coefficient at the surface in Btu/ft.^{2} hr.$^\circ$F.

T_s, T_a = Temperatures of surface and air, respectively - $^\circ$F.

The convection coefficient is not a constant, its value depending on the shape, orientation and dimensions of the surface, the physical properties of air, the nature of its flow, and its velocity, as well as on temperature.

When convection is caused by a change in the temperature and density of the air, it is called natural convection. If it is caused by an induced movement of air (e.g. by the use of a fan) it is referred to as forced convection. In practice both modes contribute in varying degrees to the convection heat transfer process.

The natural convection coefficient for a surface may be expressed in dimensionless terms by Nusselt, Grashof and Prandtl numbers as follows:[21,22]

---

21. Griffith, E., and Davis, A.H., "Transmission of heat by radiation and convection". Food Investigation Board, Special report No.9, H.M.S.O. London, 1922.

22. Min, T.C., Schutrum, L.F., Parmelee, G.V., and Vouris, J.D., "Natural convection and radiation in a panel-heated room". Heating, Piping and Air Conditioning Journal, May 1956.

$$N_{Nu} = C(N_{Gr}.N_{Pr})^{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.29)$$

where

C, d = constants determined from experiment.

N_{Nu} = Nusselt number = $\frac{h_{nc}.L}{k}$

N_{Gr} = Grashof number = $\frac{g.B.L^3.\rho^2 \triangle T}{\mu^2}$

N_{Pr} = Prandtl number = $\frac{c\mu}{k}$

and where,

h_{nc} = coefficient of natural convection.

k = thermal conductivity of air.

L = characteristic length of surface.

g = acceleration due to gravity,

B = coefficient of expansion of air.

ρ = specific weight of air.

c = specific heat of air

μ = absolute fluid viscosity of air.

$\triangle T$ = Temperature difference between air and surface.

Equation (1.29) gives:

$$h_{nc} = C.\ \frac{k}{L} \left(\frac{g.B.L^3.\rho^2.c.\triangle T}{\mu k}\right)^{d} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.30)$$

In the laminar flow region (where the product $N_{Gr}.N_{Pr}$ is between 10^5 and 10^8) Billington[23] gives

---

23. Billington, N.S., "Building Physics - Heat". Pergamon Press, London, 1967.

$$N_{Nu} = 0.54\,(N_{Gr} \cdot N_{Pr})^{\frac{1}{4}} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.31)$$

and for the turbulent region, $(N_{Gr} \cdot N_{Pr} > 10^9)$:

$$N_{Nu} = 0.13\,(N_{Gr} \cdot N_{Pr})^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots \quad (1.32)$$

It will however be seen from equation (1.30) that if the physical properties of air (B, ρ, c, μ, k), are assumed constant for a particular temperature, the natural convection coefficient (h_{nc}) will depend on the temperature difference (ΔT) and the characteristic length (L).

Billington* gives the simpler relationship in the laminar flow region for a vertical surface

$$h_{nc} = 0.27\,\left(\frac{\Delta T}{L}\right)^{\frac{1}{4}} \text{ Btu/ft.}^2\text{ hr. }^{\circ}\text{F} \quad \ldots \quad (1.33)$$

and in the turbulent region

$$h_{nc} = 0.30\,(\Delta T)^{\frac{1}{4}} \text{ Btu/ft.}^2\text{ hr. }^{\circ}\text{F} \quad \ldots \quad (1.34)$$

For a horizontal surface the coefficient would have a higher or a lower value than that for the vertical surface according to whether the heat flow is upward or downward. For upward heat flow it will be one-third more. For downward heat flow one-third less.

For forced convection Parmelee and Huebscher[24] suggested the expressions

$$(N_{St})\,(N_{Pr})^{\frac{2}{3}} = \frac{0.664}{(N_{Re})^{0.5}} \quad \ldots\ldots\ldots\ldots \quad (1.35)$$

for laminar flow, and

$$(N_{St})\,(N_{Pr})^{\frac{2}{3}} = \frac{0.2275}{(\log N_{Re})^{2.58}} \quad \ldots\ldots \quad (1.36)$$

for turbulent conditions

---

* Ibid.

24. Parmelee, G.V., and Huebscher, R.G., "Forced convection heat transfer from flat surfaces". A.S.H.V.E. Research Bulletin, Vol. 53, No.3, 1947.

where,

$$N_{St} = \text{Stanton number} = \frac{h_{fc}}{\rho c v}$$

$$N_{Pr} = \text{Prandtl number} = \frac{c\mu}{k}$$

$$N_{Re} = \text{Reynolds number} = \frac{\rho L v}{\mu}$$

and where,

h_{fc} = coefficient of forced convection.

v = fluid velocity.

and all other terms as defined for equation (1.30).

From equation (1.35) for laminar flow:

$$h_{fc} = \frac{0.664\, c \rho v}{(N_{Re})^{0.5} (N_{Pr})^{\frac{2}{3}}} \quad \text{..........} \quad (1.37)$$

and from equation (1.36) for turbulent flow:

$$h_{fc} = \frac{0.2275\, c \rho v}{(\log N_{Re})^{2.58} (N_{Pr})^{\frac{2}{3}}} \quad \text{....} \quad (1.38)$$

At the outside surface of a building, under high wind speeds, forced convection may assume a greater significance in the convection heat transfer in relation to natural convection. At low wind speeds, however, forced convection will be augmented by natural convection.

Having estimated the natural and forced components of the convection coefficient, these are added* together to obtain the total convection coefficient for heat transfer at the surface (h_c), so that:

$$h_c = h_{fc} + h_{nc} \quad \text{...............} \quad (1.39)$$

---

* It has been argued by Roux (see footnote 1.4) that this is the most reasonable procedure in the absence of contrary experimental evidence.

Denoting the convection coefficient at the outside surface of a building element by (h_{co}), we have equation (1.28) written for convection heat exchange at the outside surface:

$$w_3 = h_{co} (T_{so} - T_o) \quad \text{................} \quad (1.40)$$

where,

w_3 = net heat transfer by convection from the outside surface to the outdoor air per unit time per unit area of surface.

T_{so}, T_o = The outside surface, and the outdoor air temperatures, respectively.

1.1.4.4. Moisture

Heat may be transferred to or from the surface of a building element by the condensation or evaporation of moisture, as latent heat is drawn from, or released to the surface. Heat may also be transported through the material by internal redistribution of moisture. These effects are, however, difficult to quantify and are not significant in a hot dry climate, so that they will be neglected.

1.1.4.5. Conduction through the building material

The last factor affecting heat exchange at the surface of a building element is the conduction of heat to, or away from, the surface through the building material. The heat flux through the outside surface is obtained from equation (1.1) as:

$$w_4 = - k \frac{\partial T_{so}}{\partial x} \quad \text{..................} \quad (1.41)$$

where,

w_4 = Heat transfer through the surface per unit area of surface per unit time.

k = conductivity of material at the surface.

$\frac{\partial T_{so}}{\partial x}$ = temperature gradient at the surface.

1.1.4.6. Heat balance at the outside surface:

The heat balance at the outside surface of a building element can now be expressed by equating the net heat gain by the surface to the heat conducted away through the surface, i.e.

$$w_1 - w_2 - w_3 = w_4 \quad \ldots\ldots\ldots\ldots\ldots \quad (1.42)$$

where w_1, w_2, w_3 and w_4 are given by equations (1.14), (1.25), (1.40) and (1.41) respectively, so that the heat balance equation is:

$$\alpha I - h_{ro}(T_{so}-T_e)-h_{co}(T_{so}-T_o) = -k\frac{\partial T_{so}}{\partial x} \quad \ldots\ldots\ldots\ldots (1.43)$$

1.1.5. Heat exchange at the inside surface of a building element.

At the inside surface of a building element, which is not directly exposed to the outside environment, heat exchange takes place by:

(1) convection between the surface and the indoor air.

(2) long wave radiation between the surface and other surfaces that it 'sees'.

1.1.5.1. Convection

A general discussion of the factors involved in the convection process was given in section (1.4) which applies for convection heat transfer at the inside surface. Using the correct notation for the inside surface, we have the equivalent of equation (1.40) as:

$$w_5 = h_{ci} (T_{si} - T_i) \quad \ldots\ldots\ldots\ldots\ldots \quad (1.44)$$

where,

w_5 = net heat transfer by convection from the inside surface to the indoor air per unit time per unit area of surface.

T_{si}, T_i = Inside surface and indoor air temperatures, respectively.

h_{ci} = convection coefficient at the inside surface as given by equation (1.39).

1.1.5.2. Long wave radiation

The net heat exchange by long wave radiation between two sides of an enclosure that 'see' one another is given by:[25]

$$q_{1-2} = \sigma F_e F_{1-2} A_1 (\tau_1^4 - \tau_2^4) \quad \ldots \quad (1.45)$$

where,

q_{1-2} = net heat exchange by long wave radiation between surfaces (1) and (2) per unit time, Btu/hr.

σ = Stephan-Boltzmann constant = 0.173×10^{-8} Btu/ft.^{2}hr.^{0}R^4.

F_e = emissivity factor depending on the emissivities of the two surfaces, their shape and relative position to one another, dimensionless.

F_{1-2} = shape factor representing the fraction of radiation emitted by surface (1) falling on surface (2), dimensionless.

---

25. Brown, A.I., and Marco, S.M. "Introduction to heat transfer" McCraw-Hill Book Co., N. York, 1951.

A_1 = area of surface (1), ft^2

τ_1, τ_2 = absolute temperatures of surfaces (1) and (2) respectively in $^{\circ}R$.

This assumes that the surfaces are opaque to long wave radiation and that they emit diffusely. The emissivities are assumed high enough to make reflection negligible. The intervening media between the surfaces are assumed non-absorbing.

The net exchange can also be expressed in terms of a radiation coefficient ($h_{r(1-2)}$) as follows:

$$q_{1-2} = h_{r(1-2)}(T_1 - T_2) \cdot A_1 \quad \ldots\ldots\ldots\ldots\ldots (1.46)$$

where,

$h_{r(1-2)}$ = coefficient of radiative heat exchange between surfaces (1) and (2) based on the area (A_1) of surface (1) in Btu/ft.^{2} hr.$^{\circ}$F.

T_1, T_2 = temperatures of surfaces (1) and (2), respectively, in $^{\circ}$F.

From equation (1.45) and (1.46) we have

$$h_{r(1-2)} = \frac{\sigma F_e F_{1-2}(\tau_1^4 - \tau_2^4)}{T_1 - T_2}$$

or

$$h_{r(1-2)} = \sigma F_e F_{1-2}(\tau_1 + \tau_2)(\tau_1^2 + \tau_2^2) \quad \ldots\ldots\ldots\ldots (1.47)$$

The emissivity (F_e) is given by Hottel[26] as follows:

---

26. Hottel, H.C. "Radiant heat transmission". Mechanical Engineering, Vol. 52, No. 7, July, 1930.

For two parallel and opposite rectangular surfaces where the areas are large compared to the distance between the surfaces:

$$F_e = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad \ldots\ldots\ldots\ldots\ldots \ (1.48)$$

and for two rectangular areas with common sides, in perpendicular planes

$$F_e = \epsilon_1 \cdot \epsilon_2 \quad \ldots\ldots\ldots\ldots\ldots \ (1.49)$$

where,

ϵ_1, ϵ_2 = emissivities of surfaces (1) and (2).

The shape factor (F_{1-2}) is given by:[27]

$$F_{1-2} = \frac{1}{A_1} \int_{A_1 A_2} \frac{\cos\phi_1 \cos\phi_2}{\pi l_{1-2}^2} \, dA_1 dA_2 \ \ldots\ldots\ldots\ldots \ (1.50)$$

where, ϕ_1, ϕ_2 = angles through which the elemental areas dA_1 and dA_2 'see' one another measured from the normal to each area.

l_{1-2} = distance between surfaces (1) and (2).

The shape factor has the following characteristics:*

(a) by reciprocity,

$$F_{1-2} \cdot A_1 = F_{2-1} \cdot A_2 \quad \ldots\ldots\ldots\ldots\ldots \ (1.51)$$

(so that $h_{r(1-2)} \cdot A_1 = h_{r(2-1)} \cdot A_2$)

(b) by the property of closure the sum of all shape factors for one surface with respect to other surfaces forming an enclosure must equal unity,

i.e. $F_{1-2} + F_{1-3} + F_{1-4} \ldots\ldots + F_{1-n} = 1 \ \ldots\ldots\ldots\ldots \ (1.52)$

where n = number of surfaces 'seen' by surface (1).

---

27. Oppenheimer, A.K., "Radiation analysis by the network method". A.S.M.E. Transactions, Vol. 78, 1956.

* Ibid.

1.2. THERMAL INSULATION

1.2.1. Mechanism of thermal insulation

The main purposes of thermal insulation of buildings in hot dry climates are[28]:

1. To minimise the rate of heat flow through the structure, and maintain a temperature difference between the indoor and outdoor environments.

2. To improve thermal conditions for occupants, by suppressing directional radiation from warm building elements.

3. To control thermal movements in structural elements, by modifying thermal gradients.

4. To ensure a quick response to intermittent summer cooling (or winter heating) of heavy structures.

Thermal insulation materials may depend on their low thermal conductivity or on their surface properties of low absorptivities and emissivities to radiation. The first kind is usually referred to as bulk insulation, the second as reflective insulation.

The low thermal conductivity of most bulk insulation materials is due to the air trapped in the fabric[29, 30], hence the observed variation of conductivity with density. Although, in general, the lighter the material the lower the conductivity, materials of the same density may sometimes have

---

28. van Straaten, J.F., "Thermal performance of buildings". Architectural Science Series, Elsevier Publishing Co., London 1967.
29. Rowely, F.B., Jordan, R.C., Lund, C.E., and Lander, R.M., "Gas is an important factor in the thermal conductivity of most insulating materials". A.S.H.V.E. Transactions, Vol. 58, 1952.
30. Lander, R.M., "Gas is an important factor in the thermal conductivity of most insulating materials - Part II", A.S.H.V.E. Transactions, Vol.61, 1955.

different conductivities due to structural differences regarding pore size, distribution and interconnection. The thermal conductivity of still air (unless the pores are filled with a different gas) is, therefore, the lower limit to the thermal conductivities of porous and fibrous materials. This low conductivity (0.16 Btu in./ft.^{2}hr.$^\circ$F) is not, however, reached in practice owing to the existance of radiation and microconvection currents in the pores.

The thermal conductivity also varies with temperature*, in general increasing slightly with increase in temperature. The change in conductivity encountered in the temperature range usual in buildings is, however, only of a small order.

The effect of moisture on conductivity is more pronounced. Since water has a thermal conductivity (4.2 Btu in./ft.^{2}hr.$^\circ$F) about twenty five times higher than that of still air, the replacement of air by moisture in the insulating fabric increases the thermal conductivity appreciably. Pratt[31] has, however, reported the finding that a rapid rise in the conductivity of porous inorganic materials occurs with the first few per cent moisture content, then the rate of change of conductivity diminishes with further increase in moisture content and becomes practically constant.

Reflective insulation materials depend on the surface properties of low absorptivity and emissivity to retard heat flow due to radiation. Such materials are usually polished metal foils. In this category should, probably, also be included light paints and surface finishes.**

---

* Ibid.

31. Pratt, A.W., "Some observations on the variation of the thermal conductivity of porous inorganic solids with moisture content". Research series 30, current papers, B.R.S., D.S.I.R., 1964.

** The effect of these may depend on the wavelength of radiation.

The main problem, concerning reflective insulation, is the deterioration of the surface finish. Drastic reductions in the insulation values may take place due to discolouration and dust accumulation, which are common problems in hot dry climates.

1.2.2. Types of thermal insulation

Insulating materials can be classified in different ways. According to origin* four types can be identified:

1. Animal extracts, such as wool or hair which have limited use in building.
2. Mineral extracts, such as asbestos, glass wool and metal foils.
3. Artificial compounds, such as foam plastics like polystyrene.
4. Vegetable extracts, such as straw, cotton or cork.

According to the form in which they come, insulating materials may be grouped as follows:

1. Loose-fill materials, which come in loose form like granulated cork or exfoliated vermiculite.
2. Blanket or sheet formed materials, such as fibre glass batts and metal foils.
3. Boards and slabs, such as expanded polystyrene boards and foam concretes.
4. In-Situ formed materials, such as asbestos spray and polyurethane foam.

---

* loc. cit. see footnote 1.28

The choice of an insulating material is usually made with considerations other than the insulating properties in mind. Depending on the location of the insulation in the structure, properties such as weight, mechanical strength, appearance, durability and moisture absorption characteristics may influence the choice.

CHAPTER 2

ANALOGUE SIMULATION

2.1. THEORY

2.1.1. General

There is no scarcity of literature on analytical treatment of the thermal response of buildings, [*,1,2,3,4] Very complex procedures can, however, be involved except in the treatment of fairly simple physical systems. Alternative approaches to the study of the thermal behaviour of buildings are the investigation of conditions in actual buildings, and the use of models.

The use of actual buildings, or full scale experimental set-ups, has the advantage of eliminating errors, which may arise from change of scale or inexact reproduction of the physical system, from which models may suffer. On the other hand there is the disadvantage of the cost involved in full scale testing, the difficulty of controlling design parameters and varying them at will, and the difficulty of interpretation of the results due to the number of variables which may be involved.

---

* Loc.cit. see footnotes (1.3), (1.9), (1.10) and (1.11).

1. Mackey, C.O. and Wright, L.T. "Periodic heat flow: Composite walls and roofs." Heating, Piping and Air Conditioning, Vol. 18, No. 6, 1946.

2. Muncey, R.W. "The calculation of temperatures inside buildings having variable external conditions." Aust. J. Applied Science, Vol. 4, No. 2, 1953.

3. Pratt, A.W. and Ball, E.F., "Transient cooling of a heated enclosure." Int. J. Heat Mass Transfer, Vol. 6, 1963.

4. Raychaudhuri, B.C., "Transient thermal response of enclosures: The integrated thermal time-constant." International J. Heat Mass Transfer, Vol. 8, 1965.

An attractive alternative is the use of a model to represent the physical system. Models in use in environmental and thermal research are of two kinds:[5]

(i) Reduced scale models

(ii) Analogue models.

Reduced scale models are those which reproduce on a smaller scale the physical characteristics of the full scale system. Such models would be found in such fields, other than thermal research, as daylighting and aerodynamics.

To give reliable results such models must be dimensionally similar to the system they represent. The proper scale choice depends on the problem being investigated. If a distorted representation of the physical system is inevitable proper correction factors must be taken into account when interpreting test results.

Such models have the advantage over full scale set-ups in that they can be designed to isolate variables, and can be tested under a controlled, as well as a natural, environment.

Analogue models do not attempt to reproduce the physical characteristics of a system; but to replace it by another physical system which obeys similar laws.* Information obtained from an analogue model has to be interpreted with respect to the initial physical system.

Hydraulic and electrical analogues have been in use for the study of

---

5. Page, J.K., "Environmental research using models." Architect's J. Information Library. 11 March 1964.

* Ibid.

thermal phenomena. A hydraulic analogue[6,7] makes use of the similarities of pressure distribution and storage of fluids in pipes of various lengths and diameters, and the temperature distribution, and heat storage, in building elements.

Electric analogues[8] make use of the distribution of electric potential in a resistor-network, and the storage of electric charge in capacitors, to simulate temperature distribution and heat storage in the thermal system.

Whereas reduced scale models do no necessarily require a knowledge of the laws governing the physical system they represent, an analogue model can only be based on a prior knowledge of these laws.*

2.1.2. Thermal-electrical analogy.

The mathematical laws that govern the flow of heat and the flow of electric current are analogous.

The heat flux (w) in the steady state for heat flow in one direction was given by equation (1.2.) as

$$w = k \frac{T_1 - T_2}{l}$$

---

6. Leopold, C.S., "Hydraulic analogue for the solution of problems of thermal storage, radiation, convection and conduction." A.S.H.V.E. Trans. Vol. 54., 1948.

7. Mackey, C.O. and Gay, N.R., "Cooling load from sunlit glass." A.S.H.V.E. Trans. Vol. 58, 1952.

8. Paschkis, Victor, "Periodic heat flow in Building walls determined by electrical analogy method." Heating, piping and air conditioning, Feb. 1942.

* Loc. cit. See footnote (2.5).

which can be written

$$T_1 - T_2 = w. \frac{1}{k}$$

(Temperature difference = Heat flux x thermal resistance)

This is the same as Ohm's law:

$$V = iR \text{ .... } (2.1.)$$

(Potential difference = Current x Electric resistance)

The thermal energy (Q_t) stored in a homogeneous solid is given by:

$$Q_t = C_t . \Delta T \text{ .... } (2.2.)$$

(Heat stored = Thermal capacity x Temperature difference) and the electric charge (Q_e) stored in a capacitor is given by

$$Q_e = C_e . V \text{ .... } (2.3.)$$

(Electric charge = Electric capacitance x Potential difference)

The general Fourier conduction equation for steady state heat flow in three dimensions,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \text{ ....} (2.4.)$$

is analogous to the Laplace equation for steady current flow in a conductor in three dimensions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \text{ .... } (2.5.)$$

For unsteady heat flow equation (1.11) gives the temperature distribution as

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k} \frac{\partial T}{\partial t}$$

The distribution of electric potential produced by a current flowing through an element of distributed resistance (R) and capacitance (C), and where there is no inductance or leakage is:*

---

*Loc. cit. see footnote (1.2).

$$\frac{\partial^2 V}{\partial x^2} = RC \frac{\partial V}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.6.)$$

where,

R = resistance per unit length.

C = capacitance per unit length.

In both thermal and electrical terms the resistance and capacitance of an element are dimensionally related by the following equation

Resistance x Capacitance = Time

or, $R.C = t$ .... (2.7.)

This is borne out by dimensional analysis. Thermally:

Resistance $\left(\frac{{}^{o}F.hr.}{Btu}\right)$ x Capacitance $\left(\frac{Btu}{{}^{o}F}\right)$ = Time (hr.)

Electrically:

Resistance (ohm = $\frac{volt}{amp}$) x Capacitance (farad = $\frac{amp.sec.}{volt}$) = Time (sec.)

These similarities lead to a scheme of equivalent thermal and electrical quantities as shown in Table 2.1. .

Table 2.1.

Thermal-electrical equivalence

Thermal			Electrical		
Item	Units	Symbol	Item	Units	Symbol
Heat	Btu	Q_t	Charge	Coulomb	Q_e
Heat flow	Btu/hr.	q	Current	$\frac{Coulomb}{sec.}$ = ampere	i
Temperature	^{o}F	T	Potential	Volts	E
Resistance	^{o}F.hr/Btu	R_t	Resistance	$\frac{Volt}{amp.}$ = Ohm	R_e
Capacity	Btu/^{o}F	C_t	Capacitance	$\frac{Coulomb}{volt}$ = Farad	C_e
Time	hour	t_t	Time	Second	t_e

It is, therefore, possible to replace a thermal system with an analogous electrical resistance-capacitance network. For the analogy to exist the following conditions must be met:[9]

(i) Dimensionless moduli characterising the electrical circuit must be equal to those characterising the thermal circuit.

(ii) Electrical and thermal circuits must be schematically similar.

(iii) The boundary conditions imposed on the electrical circuit must be identical to those that exist for the thermal system, and must be correctly related in time.

The first of these conditions is the basis for the quantitative relationship between the two analogous systems, and leads to a consideration of the scale factors necessary for conversion from one system to the other.

The second condition leads to the question of how the parameters of the circuit representing the thermal system can be reproduced in the electrical circuit.

The third poses the problem of simulating electrically, the boundary conditions obtaining for the thermal system.

These three points will now be dealt with in turn.

---

9. Buchberg, H., "Electric analogue prediction of the thermal behaviour of an inhabitable enclosure." A.S.H.R.A.E. Trans. Vol. 61, 1955.

2.1.3. Scaling.

Using the notation of Table 2.1, if ratios of the equivalent electrical and thermal items in the two analogous systems can be defined as follows:

$$N_Q = \frac{Q_e}{Q_t} \quad \text{(Quantity)} \quad \ldots\ldots\ldots\ldots \quad (2.8)$$

$$N_q = \frac{i}{q} \quad \text{(Flow)} \quad \ldots\ldots\ldots\ldots \quad (2.9)$$

$$N_E = \frac{E}{T} \quad \text{(Potential)} \quad \ldots\ldots\ldots\ldots \quad (2.10)$$

$$N_R = \frac{R_e}{R_t} \quad \text{(Resistance)} \quad \ldots\ldots\ldots\ldots \quad (2.11)$$

$$N_C = \frac{C_e}{C_t} \quad \text{(Capacitance)} \quad \ldots\ldots\ldots\ldots \quad (2.12)$$

$$N_t = \frac{t_e}{t_t} \quad \text{(Time)} \quad \ldots\ldots\ldots\ldots \quad (2.13)$$

then the six (N) variables are the scale factors relating the electrical and thermal circuits quantitatively. Scale factors must satisfy the following relationships[10]

(i) Defining equation of Quantity:

$$N_Q = N_q \cdot N_t \quad \ldots\ldots\ldots\ldots \quad (2.14)$$

(ii) Ohm's law:

$$N_E = N_q \cdot N_R \quad \ldots\ldots\ldots\ldots \quad (2.15)$$

(iii) Equivalence of time constants:

$$N_t = N_R \cdot N_C \quad \ldots\ldots\ldots\ldots \quad (2.16)$$

Up to three of the six variables in equations (2.14-16) may be assigned any convenient values (as long as the three do not occur in one equation), the numerical values of the rest must follow from the given relationships (2.14-16).

---

10. Nottage, H.B., and Parmelee, G.V. "Circuit analysis applied to load estimating." A.S.H.R.A.E. Trans. Vol. 60, 1954.

The actual values chosen for the scale factors in this work will be discussed in section 2.2.5.

2.1.4. Lumping circuit parameters

There is a difference of property between the thermal system and the electrical network that represents it.[11] A building element has, essentially, a distributed resistance and heat storage capacity. The electrical circuit that simulates the element is composed of discrete resistances and capacitances. The distributed property system is, therefore, seen as being composed of sections or lumps with concentrated resistance and capacity. The lumped system will approach the distributed property system as the size of lumps becomes smaller and the number of lumps approaches infinity.

It has, however, been shown[*] that adequate accuracy can be obtained by the use of a finite number of lumps to represent each particular building element.

The conduction path in a building element can be represented by a series of pure lumped capacitances interconnected by pure lumped resistances. These are appropriately related to be schematically similar to the thermal equivalent.

The value of the conduction path resistances and capacitances is determined as follows:[*]

---

11. Buchberg, H., "Electric analogue studies of single walls." A.S.H.R.A.E. Trans. Vol. 62, 1956.

* Ibid.

By definition: thermal resistance is the driving potential (temperature difference) per unit heat flux.

$$R_t = \frac{T_1 - T_2}{q} \quad {}^{\circ}F.hr/Btu \quad .... \quad (2.17)$$

From equation (1.2), the heat flux for a heat transfer area (A), is

$$q = kA\frac{T_1 - T_2}{l}$$

$$\text{Therefore } R_t = \frac{l}{k.A} \quad {}^{\circ}F.hr/Btu \quad .... \quad (2.18)$$

where (k) is the conductivity and (l) is the thickness of the conduction path.

The electrical resistance (R_e) is then given by

$$R_e = R_t.N_R \quad \text{ohms} \quad .... \quad (2.19)$$

where N_R, a scale factor, is given by equation (2.11)

The thermal capacity is by definition the heat necessary to cause unit change in temperature of mass involved,

$$C_t = \frac{Q_t}{T_1 - T_2} \quad Btu/{}^{\circ}F \quad .... \quad (2.20)$$

But

$$Q_t = \int_{T_2}^{T_1} \rho.c.l.A.dT$$

where ρ = density

c = specific heat

$l.A$ = volume of mass involved.

If the density (ρ) and specific heat (c) are constant over the temperature interval ($T_1 - T_2$), Q_t may be written as

$$Q_t = \rho clA(T_1 - T_2) \quad Btu \quad .... \quad (2.21)$$

from equation (2.20) and (2.21) it follows that the thermal capacity (C_t) is:

$$C_t = \rho clA \quad .... \quad Btu/{}^{\circ}F \quad ..... \quad (2.22)$$

and the electrical capacitance (C_e) is then

$$C_e = C_t \cdot N_C \text{ farads} \quad \ldots\ldots\ldots\ldots (2.23)$$

where N_C, a scale factor, is given by equation (2.12).

2.1.5. Simulation of boundary conditions

Significant factors entering into the heat exchange at the boundary surface of a building element were given in the previous chapter (section 1.4) for the outside surface and (section 1.5) for the inside surface.

For the outside surface, these are:

(a) absorbed solar radiation.

(b) long wave radiation exchange between surface and environment.

(c) convection exchange between surface and outdoor air.

and for the inside surface:

(a) Long-wave radiation exchange between the inside surface and other surfaces that it 'sees'.

(b) convection exchange between surface and indoor air.

2.1.5.1. Solar radiation

Since the sun radiates at a very high temperature the radiation exchange between an exposed surface and the sun is essentially independent of the temperature of the surface, and is, therefore, a pure heat input.[12] The heat flux due to solar radiation was given by equation (1.14) as

---

12. Buchberg, H., "Circuit analysis applied to solar house-heating". Transactions of the Conference on the use of Solar Energy, Tucson, Arizona, U.S.A., Nov. 1955 - Standard Research Institute.

$$w = \frac{q}{A} = \alpha I$$

which gives the magnitude of the heat input for a surface of area (A) as

$$q = \alpha IA$$

In the electrical circuit this will be represented by an electric current (i), the magnitude of which, at any time, is:

$$i = \alpha IA.N_q \quad \text{amperes} \quad \ldots\ldots\ldots\ldots \quad (2.24)$$

where N_q, a scale factor, is given by equation (2.9).

To simulate this time-variable input the analogue would have a device capable of delivering a controlled time-variable current, (see section 2.2.2.4.).

2.1.5.2. Long wave radiation exchange

Long wave radiation exchange is, by Stephan-Boltzmann law, proportional to the difference of the fourth power of the temperatures of bodies involved. This exchange has, however, been given in linearised form, for the outside surface, by equation (1.25) as

$$w = \frac{q}{A} = h_{ro}\,(T_{so} - T_e)$$

and for the inside surface for an element which is part of an enclosure, as the exchange between two surfaces (1) and (2) which 'see' one another, by equation (1.46) as

$$q_{1-2} = h_{r(1-2)}\,(T_1 - T_2).A_1$$

These two equations can be written in the form;

$$\frac{1}{h_{ro}.A} = \frac{T_{so} - T_e}{q}$$

$$\frac{1}{h_{r(1-2)}.A_1} = \frac{T_1 - T_2}{q_{1-2}}$$

which by definition (equation 2.17) makes the expressions on the left-hand-side, radiative resistances, so that if the long wave radiation coefficient of heat transfer at any surface can be written as (h_r) the radiative resistance $(R_{t(r)})$ is given by

$$R_{t(r)} = \frac{1}{h_r A} \quad {}^{o}F.hr/Btu \quad \dots\dots\dots\dots \quad (2.25)$$

where (A) is the area of the surface, and the electrical resistance $(R_{e(r)})$, is given by

$$R_{e(r)} = R_{t(r)} \cdot N_R \quad \text{ohms} \quad \dots\dots\dots\dots \quad (2.26)$$

where N_R, a scale factor, is given by equation (2.11).

A potential difference, corresponding to the temperature difference, will exist in the analogue across the resistance $(R_{e(r)})$.

A controlled, time-variable potential is generated to represent the environmental temperature (T_e), (see section 2.2.2.3). The magnitude of this potential (E) is given, at any time, by:

$$E = T_e \cdot N_E \text{ volts} \quad \dots\dots\dots\dots\dots\dots \quad (2.27)$$

where N_E, a scale factor, is given by equation (2.10).

2.1.5.3. Convection exchange

The convection exchange for an outside surface, given by equation (1.40) is:

$$w = \frac{q}{A} = h_{co} \, (T_{so} - T_o)$$

and for an inside surface, by equation (1.44) as

$$w = \frac{q}{A} = h_{ci} \, (T_{si} - T_i)$$

Writing these equations in the form

$$\frac{1}{h_{co} \cdot A} = \frac{T_{so} - T_o}{q}$$

and

$$\frac{1}{h_{ci} \cdot A} = \frac{T_{si} - T_i}{q}$$

We have, by definition (equation 2.17), the expressions on the left-hand-side as convective resistances to air temperature. If a convection coefficient for any surface be written as (h_c), then the convective resistance $(R_{t(c)})$ is

$$R_{t(c)} = \frac{1}{h_c \cdot A} \quad {}^{o}F.hr./Btu \quad \ldots\ldots\ldots\ldots \quad (2.28)$$

and its electrical equivalent $(R_{e(c)})$ is

$$R_{e(c)} = R_{t(c)} \cdot N_R \quad \text{ohms} \quad \ldots\ldots\ldots\ldots \quad (2.29)$$

where N_R, a scale factor, is given by equation (2.11).

A controlled, time-variable potential is generated to represent the outside air temperature (T_o), (see section 2.2.2.3). The magnitude of this potential (E) is given, at any time, by:

$$E = T_o \cdot N_E \quad \text{volts} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.30)$$

where N_E, a scale factor, is given by equation (2.10).

A time constant potential (see section 2.2.2.5) would simulate the indoor air temperature when this is assumed constant. This potential will be

$$E = T_i \cdot N_E \quad \text{volts} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.31)$$

where N_E, a scale factor, is given by equation (2.10)

2.2. THE ANALOGUE SIMULATOR.

2.2.1. General

The experimental work connected with this thesis was carried out using the Analogue Simulator at the Department of Building Science of the University of Liverpool. An original high accuracy analogue-digital set-up, designed[13,14] for use at the Building Climatology Research Unit of the department, was given up for a less costly analogue simulator. A view of the analogue is given in Fig. 1.

The analogue has essentially three component parts:

1. Input function generators producing controlled functions for the simulation of boundary conditions.
2. Building response simulators: the networks representing a building, or building element.
3. Output measurement and recording devices.

The equipment comprising the analogue will be discussed under these categories.

2.2.2. Input function generation

2.2.2.1. Time control

The simulation of boundary conditions necessitates the generation of time-variable potential (outdoor air and environmental temperatures), time-variable current (solar radiation), as well as time-constant potential (constant indoor air temperature, etc.)

---

13. Thompson, K., and Hitchin, E.R., "Initial paper study of methods and equipment for an investigation of the thermal response of buildings to climatic variations". Unpublished report, B.C.R.U., Department of Building Science, Liverpool University, July, 1965.

14. Thompson, K., "Revised Simulator design". Unpublished report, B.C.R.U., Department of Building Science, Liverpool University, Nov., 1965.

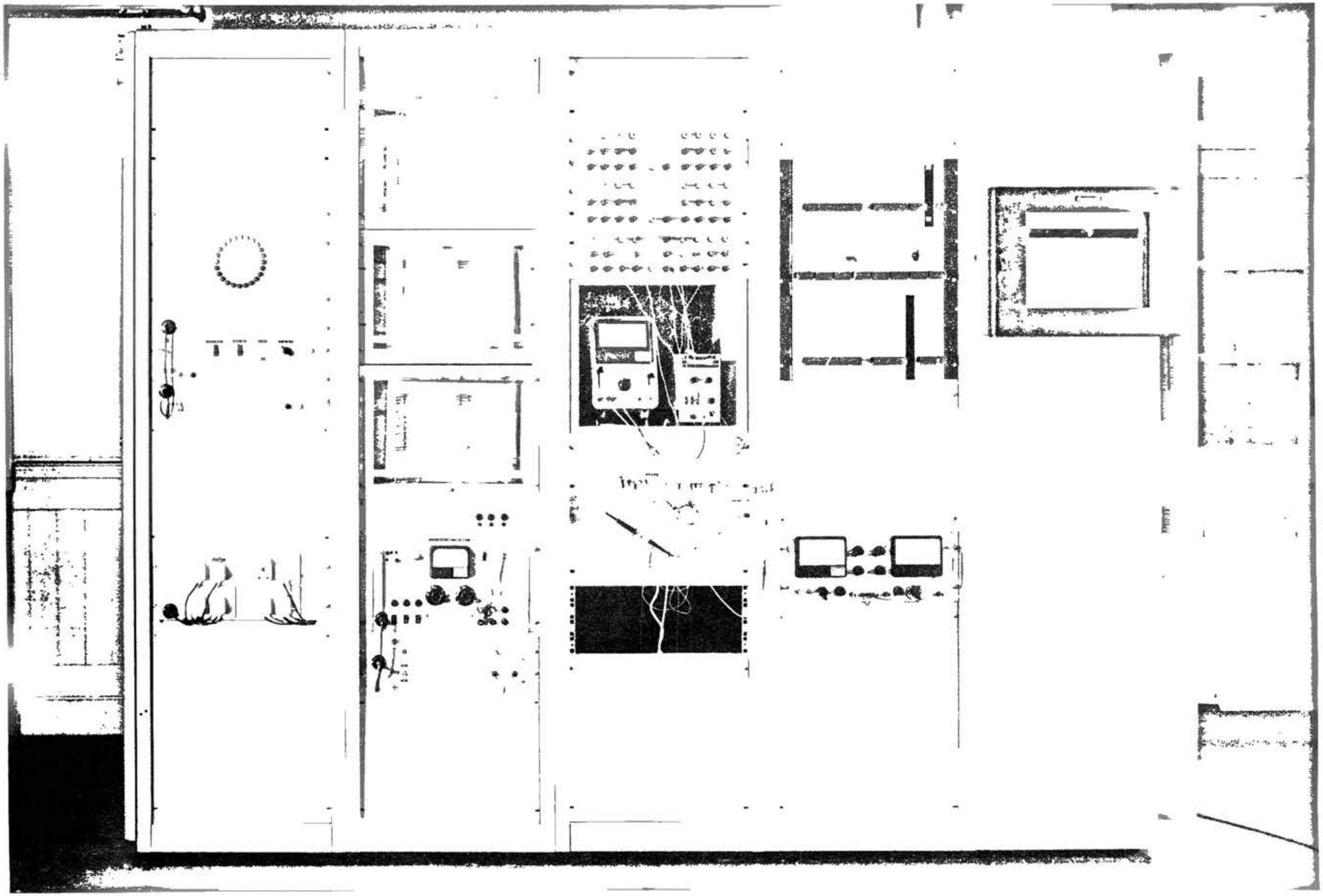


FIG. 1. GENERAL VIEW OF ANALOGUE SIMULATOR.

This requires the analogue to have a time scale and a master time reference. The analogue time scale is related to real time by the appropriate choice of the scale factor N_t (equation 2.13). The master time reference is provided by using a mains synchronous motor driving a magnetic arm which rotates over a set of 24 make-and-break reed switches arranged, equidistantly, around a circle (Fig. 2). As the magnetic arm passes over each reed switch, the switch makes, thus providing a time signal potential. This controls relays which select the appropriate time axis channel for each simulated hour, in the function generator scheme.

The time signal also operates automatic start, at preselected hours, of the output recording system. Fig. 3 shows a time-dial bulb display.

2.2.2.2. Wave form function Generators

A matrix scheme is used to generate variable potential: Two insulating Vero boards with matrix hole arrangements and conductor copper strips, are placed back to back with the insulated sides in contact, and the copper strips on one running at right angles to those on the other.

One board has forty copper strips and is used as a potential divider by mounting equal, high stability resistors, one between each two strips. A stabilised voltage is fed at one end of the potential divider while the other end is grounded.

The second board, with twenty four copper channels, provide the time axis. Each time channel is connected to the appropriate potential by inserting a brass bolt, through the hole, at the junction with the copper strip, on the potential divider, carrying that potential. This creates a visible wave form representing the diurnal variation of the simulated input.

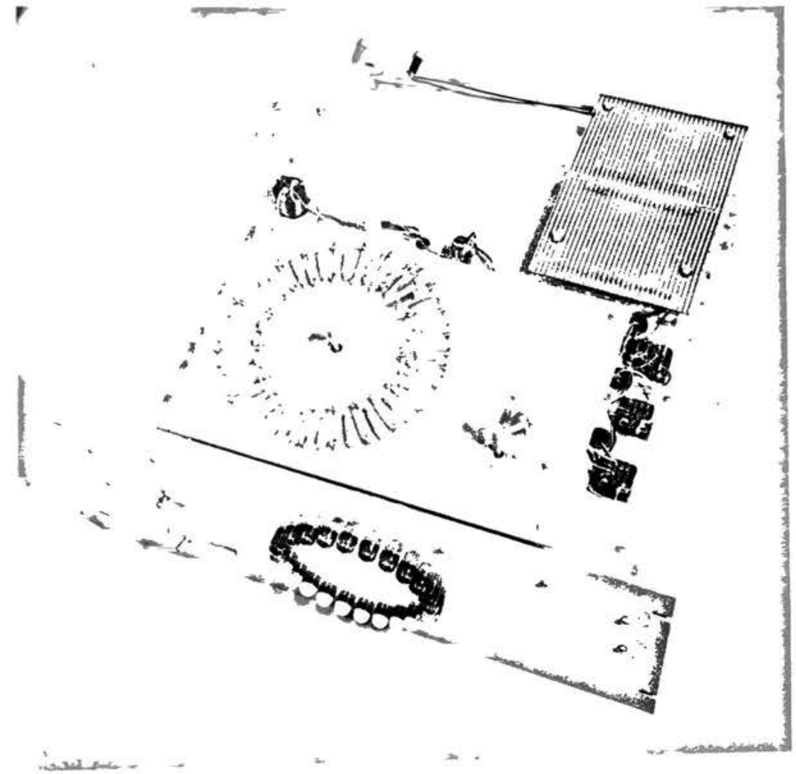


FIG. 2. TIME CONTROL UNIT.

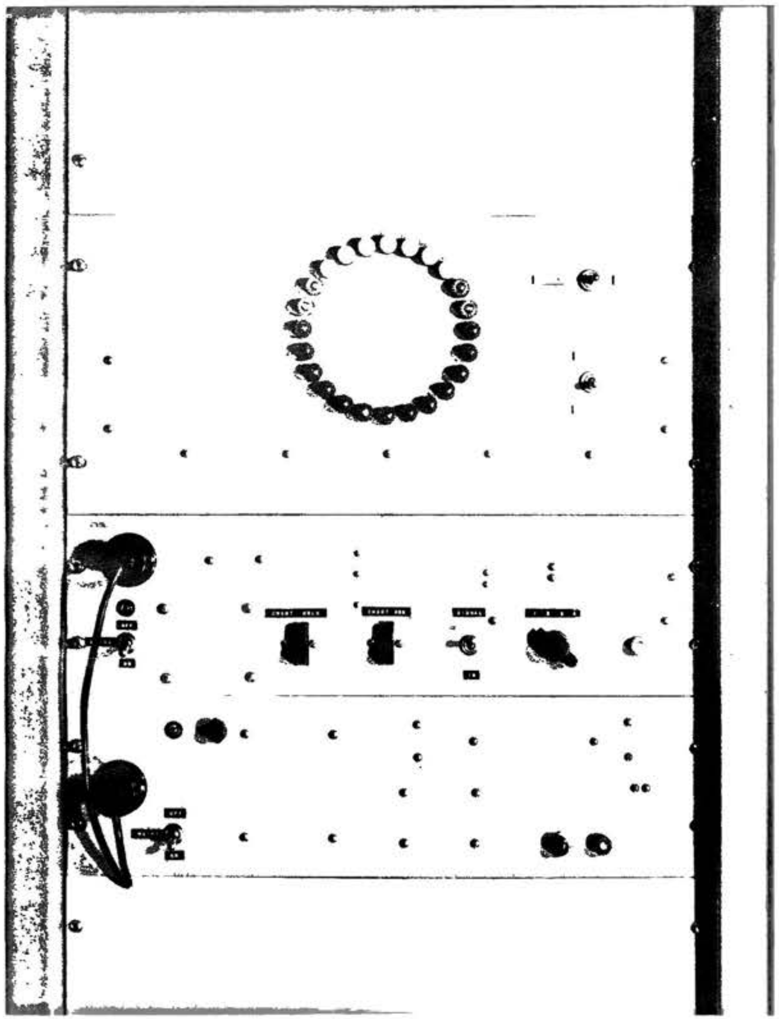


FIG. 3. TIME-DIAL DISPLAY.

As the time axis relays scan the time channels in turn, the visible wave form is reproduced in the form of a step-variable potential.

A diagrammatic representation of this system is given in Fig.4. Fig. 5 shows a view of the wave form panels.

The scheme is used to deliver variable voltage to simulate temperature sources by using a low voltage source and feeding the building response simulator networks directly. It can also be used to deliver a variable current for the simulation of heat input sources, by using a high voltage source and feeding the model networks through high resistances of appropriate values.

2.2.2.3. Variable temperature (voltage) sources:

The temperature range considered is 0-120°F. The magnitude of the voltage used depends on the choice of the scale factor N_E (equation 2.10). In practice, however, the maximum permissible voltage is determined by the voltage range of the output equipment. Buffer transistors voltage limit (see 2.2.5.) makes it desirable to stay within 10 volts. A stabilised voltage supply unit giving ± 20 volts is used.

10 ohms (± 1%) high stability resistors are used for the potential divider, the impedence of the model network is usually high enough to allow the input voltage to be fed directly to the network.

Since forty steps exist on the potential divider, the 0-120°F range is divided into steps of 3°F. It can, however, be arranged for the part of this range, which is not used in practice (e.g. 0-40°F), to be lumped into one step, by inserting the appropriate resistance (200 ohms) in the potential divider. In this case forty steps on the potential divider

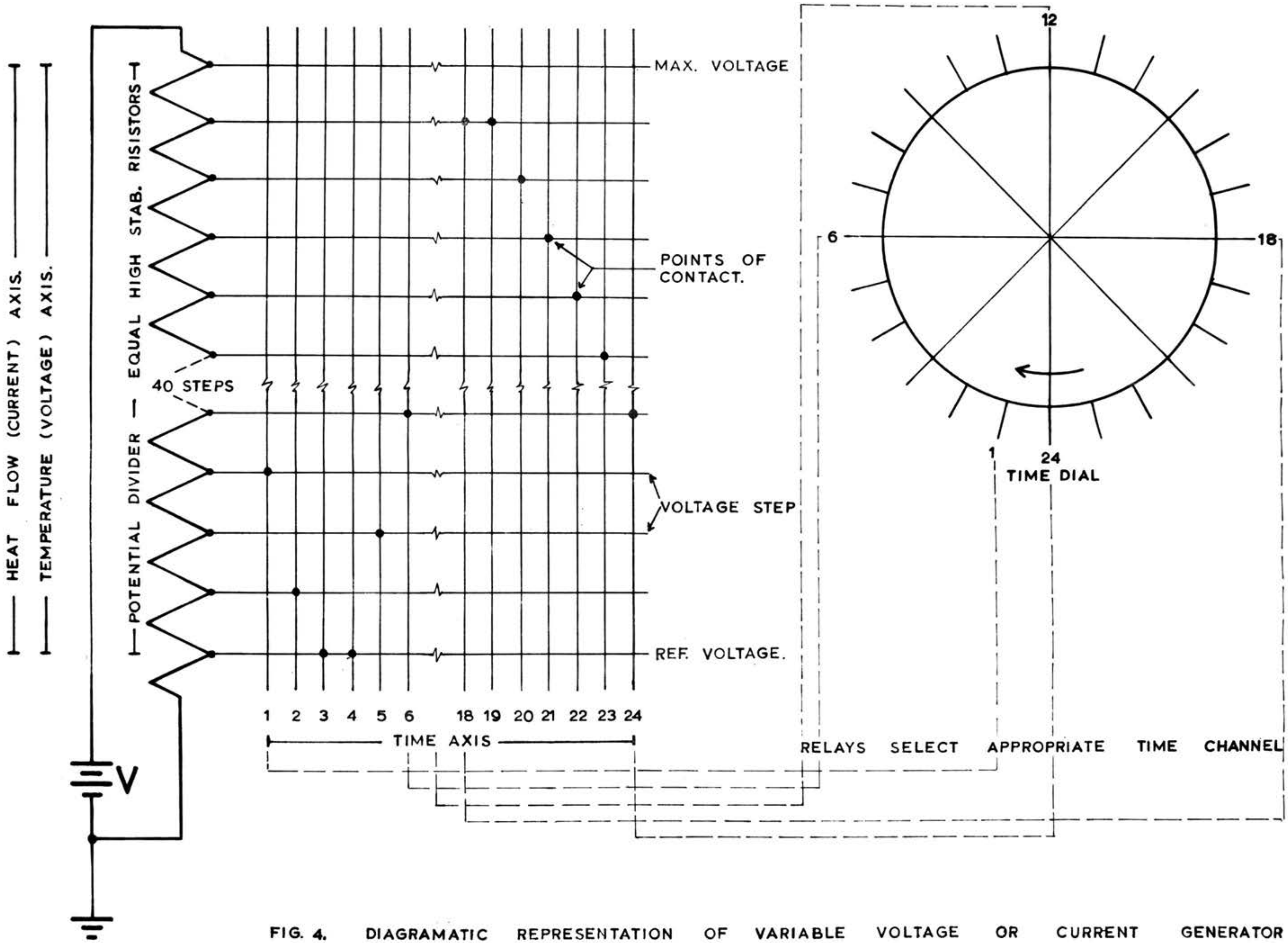


FIG. 4. DIAGRAMATIC REPRESENTATION OF VARIABLE VOLTAGE OR CURRENT GENERATOR

represent the 40-120°F range at 2°F a step.

The analogue has four variable voltage generators.

2.2.2.4. Variable heat flow (current) sources:

200 volts are applied across the potential divider which is made up of 200 ohms (± 1%) high stability resistors. The building response simulator network is fed through high resistances.

The resistance is calculated, for each input, using Ohm's Law, for a maximum current of known magnitude (see equation 2.24). In calculating the resistance the assumption is made that the model network is at reference voltage, so that the voltage drop across the resistor is 200 volts. This, in practice, may lead to an error of up to 2½%. and is not an unreasonable assumption to make.

The generator resistor (R_g) is given for any input by

$$R_g = \frac{V}{i} = \frac{V}{\alpha I_{max} A N_q} \text{ ohms} \quad \ldots\ldots\ldots\ldots (2.32)$$

where i = maximum current required (equation 2.24).

V = 200 volts.

Each step on the wave form generator is then $\frac{1}{40}$th of the peak current.

The analogue has four variable current generators.

2.2.2.5. Constant temperature (voltage) sources:

A solartron stabilised power supply unit (Type AS1410), is used for time-constant voltage inputs. The unit gives ± 30 volts in 0.1 volts steps.

2.2.3. Building response simulator networks:

The resistance-capacitance networks representing building elements are built individually. Each element is represented by a number of T-sections.

The T-network is constructed on a S.R.B.P. panel with a matrix hole arrangement. The resistor arm of the network above the board and the capacitor arm below (Fig. 6).

High stability, $\frac{1}{2}$ watt, resistors are used. An accuracy of up to $\pm$ 1% in the lower range is possible, but higher values of resistance, up to 1 megohm units, at $\pm$10% accuracy are used.

Low leakage polyester foil, 160 volts, capacitors are used. The lower accuracy limit of values is set by the strays involved in the circuits - these could be down to a few picofarads. The upper limit for capacitor values is set by physical size. Units of 1 microfarads being the largest employed, with an accuracy of $\pm$10%

2.2.4. Output measurement and recording.

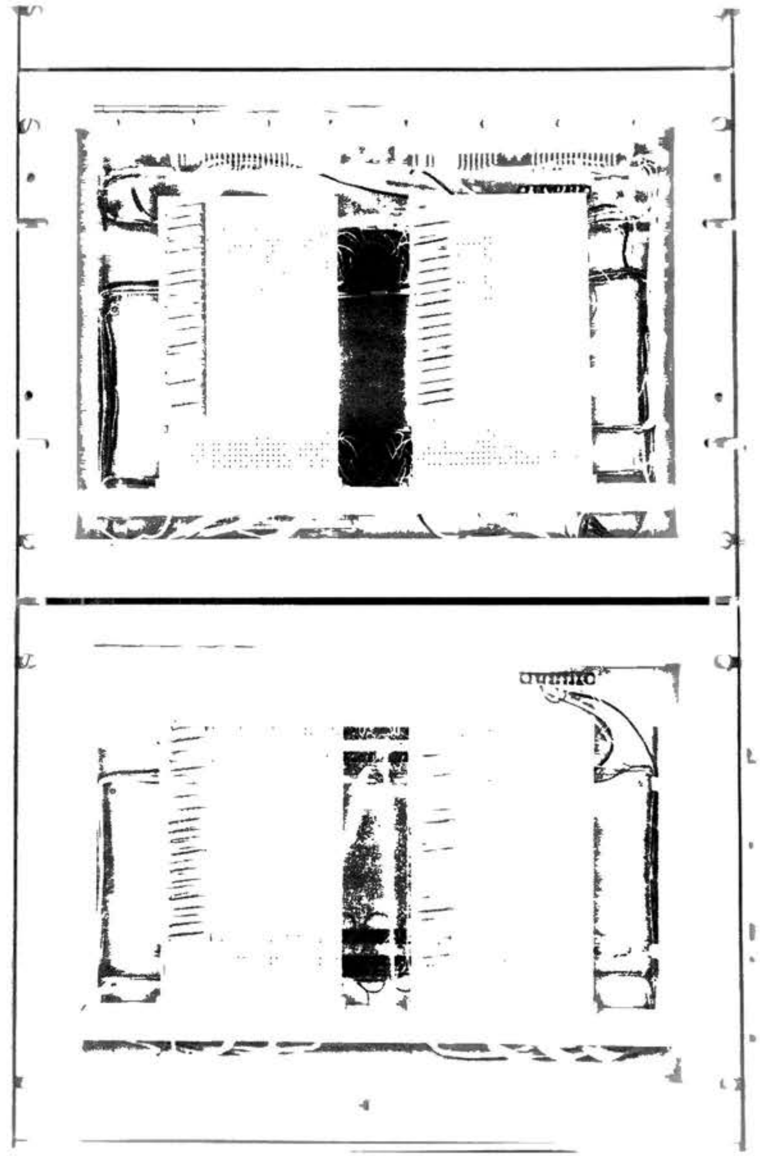
A Honeywell pen recorder is used for output recording. This has a paper speed of just over 8 inches per minute, and a full scale traverse of $\frac{1}{4}$ second, with 1% overshoot. The recorder is provided with automatic start controlled by the time signal.

The high impedence of the building simulator networks requires that a high impedence buffer be present between the tested circuit and the recorder. A unity gain, very high input impedence, buffer was designed for this purpose.[15] All output signals are fed to the pen recorder through the buffer.

An attenuator, between the buffer and the pen recorder controls the amplitude amplification of the output plot.

For spot measurements during test runs a Hewlett-Packard (Type 412A/AR) vacuum tube voltmeter was used. As a voltmeter it measures down to 1 millivolts at an accuracy of $\pm$ 1% of the full scale, and has an input

15. Thompson, K. "A very high input impedence buffer using field effect transistors". Electronic Engineering, June, 1966.



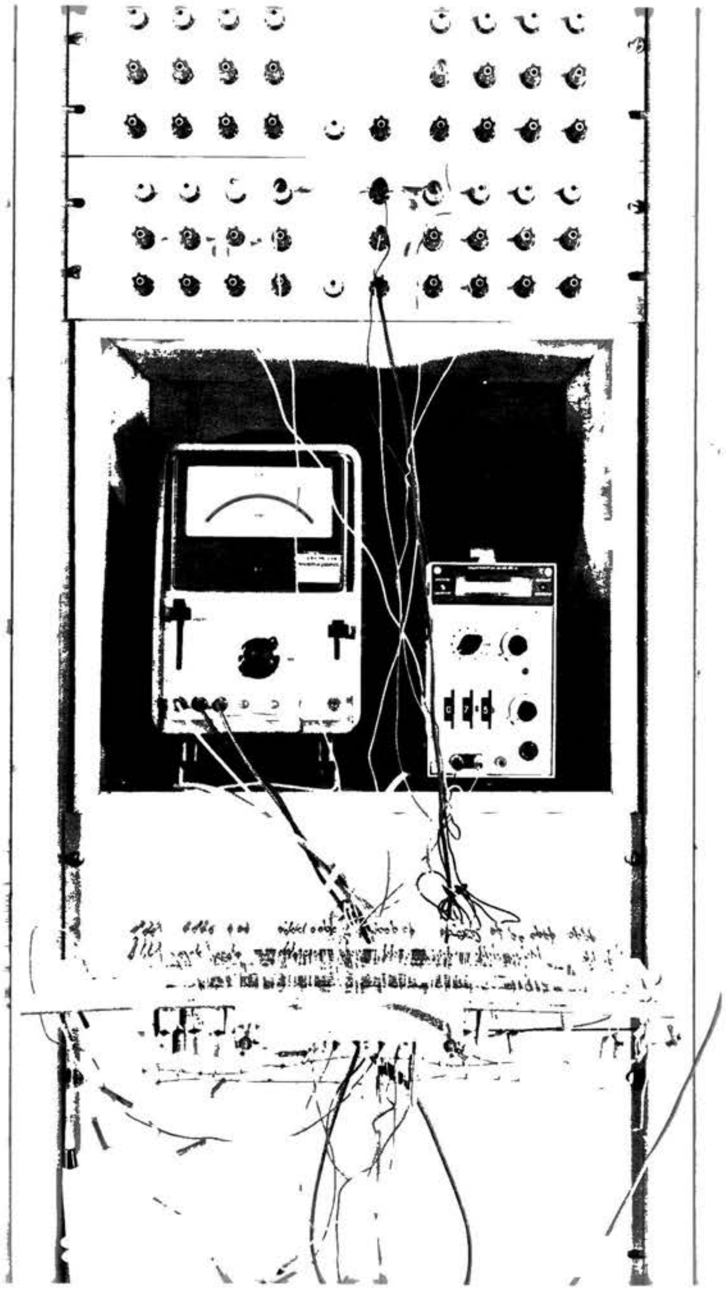


FIG. 5 WAVE FORM PANELS.

FIG. 6. MODEL NETWORK.

resistance of 200 megohms on the range 300 millivolts and above.

Used as an Ammeter it measures down to 1 microamperes at an accuracy of $\pm 2\%$ of the full scale.

2.2.5. Determination of scale factors:

The question of scaling can now be discussed. Three of the six scale factors, given by equations 2.8 - 2.13, can be assigned convenient values. The others follow from equations 2.14 - 2.16.

The choice of an operational speed, and a time scale factor, is determined by the output recording equipment. The pen recorder's just over 8 inches per minute paper speed makes the choice of a time scale factor (N_t) of one second, analogue time, to one hour real time, convenient. This gives a diurnal cycle's trace of approximately 3.3 inches.

The voltage-temperature scale factor (N_E) is governed by the requirement to be within the voltage range of the buffer transistors, which is ± 10 volts. The temperature range considered 0-120°F, which makes a scale factor of one volt to 20°F reasonable. This gives 6 volts for the full range.

The other critical choice is the scale factor (N_C) for capacitance. The problem of physical size makes units of one microfarads the highest acceptable. Considering the range of circuit parameters involved in this programme (see chapter 4), a scale factor of 10^{-8} farads to one Btu/$^{\circ}$F seems a good choice.

The other scale factors: N_R (resistance), N_q (flow), and N_Q (quantity), follow from equations 2.14-2.16. A summary is given in Table 2.2 .

Table 2.2

Summary of scale factors

Reality		Analogue		Scale factor	
Item	Value	Item	Value	Symbol	value
Time	1 hour	Time	1 second	N_t	1
Temperature	1 deg F	Voltage	0.05 volts	N_E	$\frac{1}{2} \times 10^{-1}$
Capacity	1 Btu/deg.F	Capacitance	10^{-8} farads	N_C	10^{-8}
Resistance	1 deg F.hr/Btu	Resistance	10^{8} ohms	N_R	10^{8}
Heat flow	1 Btu/hr	Current	$\frac{1}{2} \times 10^{-9}$ amperes	N_q	$\frac{1}{2} \times 10^{-9}$
Heat	1Btu	Charge	$\frac{1}{2} \times 10^{-9}$ coulombs	N_Q	$\frac{1}{2} \times 10^{-9}$

2.2.6. Accuracy

Factors affecting the accuracy of analogue predictions - apart from errors of interpretation - may be of two types:

1. Errors due to simplifications in the description of the physical system. These include approximations made in the choice of boundary design data, the choice of physical properties of building materials, and assumptions as to the nature of these materials (e.g. homogeneous, in perfect contact etc.) and the nature of the heat transfer process (e.g. unidirectional etc.).
2. Errors of analogue performance stemming from inexact reproduction of boundary conditions, lumping of the distributed properties of the physical system, component error, and inaccuracies in output recording

It is this second type that is of interest here.

The degree of accuracy expected of the different components of the analogue simulator has been indicated in the previous discussion. It is not, however, easy to estimate the contribution of individual analogue elements to the overall accuracy of predictions.

At the start, therefore, and since this experimental programme was the first to be carried out on the analogue simulator, it was decided to check the overall accuracy by comparing the analogue's solution with an analytical solution of a problem.

The problem considered was that of a wall subjected to a sinusoidal temperature variation on one side and perfectly insulated on the other.

The analytical and analogue predictions of the amplitude ratio of the temperature variations on the wall surfaces, and the time lag between the occurrence of the temperature maxima on the outside and inside surfaces, were compared.

The detailed analytical and analogue solutions are given in Appendix 1. The results are compared in Table 2.3.

Table 2.3

Item	Analytical	Analogue	% discrepancy
Amplitude ratio	0.201	0.194	$3\frac{1}{2}$%
Time lag	8.8 hrs.	9.0 hrs.	$2\frac{1}{2}$%

With reference to the analytical solution, it will be seen, that the analogue predicts the damping effect, of the building element, on the heat wave to within $3\frac{1}{2}$%, and the time lag to within $2\frac{1}{2}$%.

Although it is not suggested that the same degree of accuracy, necessarily, obtains for all other experiments carried out on the analogue. This agreement between the two predictions is, however, taken as an indication that meaningful results can be obtained using the analogue simulator.

CHAPTER 3

DESIGN DATA

3.1. WEATHER DESIGN DATA

3.1.1. General

This part deals with the selection of weather design data representative of a hot dry climate. Special reference is given to Sudan and the selection is based on Khartoum meteorological records, where available, or estimated for Khartoum.

Dry climates are the most developed over the land surface of earth, covering 26% of continental areas.[1] They prevail over North Africa, the Arabian peninsula, West and Central Australia, and the western coast of Central and South America.

The essential feature of a dry climate according to Thornthwaite,[2,3] is that potential evaporation of moisture from soil surfaces and vegetation exceeds the average annual precipitation, so that in a normal year there is a prevailing water deficiency.

Such climates are characterised by clear skies and a low water vapour content of the atmosphere. Since water vapour is a principal absorber of solar energy, and since it acts as a blanket to prevent excessive cooling of the earth's surface by virtue of its opaqueness to energy reradiated from the earth, a low atmospheric content of water vapour results in large diurnal fluctuations in temperature due to excessive heating of the earth's surface by day and

---

For a discussion on classification of climates see:

1. Trewartha, G.T., "An introduction to climate". McGraw-Hill, New York, 1954.
2. Thornthwaite, C.W., "The climates of earth". Geog. Review, July, 1933.
3. Thornthwaite, C.W., "An approach toward a rational classification of climate". Geog. Review, Jan., 1948.

excessive energy losses to the atmosphere by night. Lengthy sunshine hours and high intensities of solar radiation are usual under such climates.

3.1.2 The Sudan

The Sudan is, broadly speaking, a vast plain lying, in the tropics, between latitude 4^{o}N and 22^{o}N. Apart from the 'Sud' swamps in the south, there are no inland lakes or water surfaces large enough to produce local climatic conditions.

Satakopan,[4] using Thornthwaite's system of classification,[*] divided Sudan into four climatic zones:

1 - The Arid Zone: areas to the east of longitude 24^{o}E and north of latitude 12^{o}N. In this zone, areas south of latitude 14^{o}N (and the Red Sea Hills) have relatively higher moisture indices and are, therefore, comparatively less arid.

2 - The semi-arid zone: comprising western Darfur, the plains in Bahr El Gazal, Kordofan and Upper Nile provinces and the southern parts of the Blue Nile province.

3 - The dry sub-humid zone: comprising South Western Darfur, areas in Equatoria and Bahr El Gazal provinces where elevation is above 500 metres above sea level.

4 - The moist sub-humid zone: confined to a narrow belt along the southern border of the country in the Equatoria province.

---

4. See Bhalotra, Y.P.R. "Meteorology of Sudan". Memoir No. 6. Sudan Meteorological Service, Khartoum, 1963.

* loc. cit. see footnote 3.3.

Khartoum, the Capital and largest urban centre of Sudan, located at latitude 15°-36'N, and longitude 32°-33' E, lies in the arid sub-desert area in Northern Sudan. This area can be said to have, principally, two seasons.[5]

1 - A summer (April-October) which is hot and dry at first (April-June), becoming slightly humid and relatively less hot later (July-October).

2 - A winter (November-March) which is described in "Climate of the Sudan"[*] as a "Season of generally cool and pleasant weather". The humidity is, however, uncomfortably low. Occasionally warm spells of varying durations prevail. Less frequently, in December-February, the area is affected by cold waves arriving from the Sahara desert in the Northwest.

Meteorological averages for this area are given in Table 3.1[*].

Table 3.1.

Meteorological averages for the Khartoum area.

Item	Summer-Dry (April-June)	Summer-Wet (July-October)	Winter (November-March)
Mean Daily Max.Temp. °F	103.6	95.5	93.7
Mean Daily Min.Temp. °F	73.6	73.0	60.6
Mean Daily Range °F	30.0	22.5	33.1
Mean Relative Humidity	30%	62%	31%

Design data selected for Khartoum may, therefore, be assumed representative of a hot dry climate. It will be seen, however, that the severest weather conditions, from the thermal point of view, are likely to occur, in this area, during the early part of summer (April-June) when the highest daily maximum temperatures, the largest diurnal temperature fluctuations, the longest sunshine hours and greatest intensities of solar radiation are to be found.

5. "Climate of the Sudan". Sudan Meteorological Service, Khartoum. Undated Note.

* Ibid.

In this area winter does not pose a problem. With a mean daily minimum temperature of 60.6°F winter cannot be considered a season when the heating of buildings is necessary.

3.1.3. The Design Day

Reference to the previous discussion (section 1.4) on heat exchange at the outside surface of a building element will show the manner in which different weather factors enter into the heat exchange process.

The relevant data needed (see equation 1.43) are:

1. Hourly values of screen air temperature.
2. Hourly values of total, short wave, solar radiation falling on a surface of given orientation.
3. Hourly values of radiative sky and ground temperatures.
4. Appropriate design wind speed.

It has been suggested by Roux[6] that such design data should be chosen by a frequency analysis, of records covering a number of years, carried out for each hour of the day to find, for each weather element, a 24 hour curve such that any point in the curve is not exceeded except in a certain number of days (5% of days for air temperature and wind velocity and 10% for solar radiation).

Richards[7] suggested the use of the mean hourly values, of the weather element, for the twenty hottest days (days with highest maximum air temperatures) in each year, over a suitable number of years.

---

6. Roux, A.J.A., "A method of expressing climatic data and the use of these data in the design of buildings". Series D.R.093. N.B.R.I. Pretoria.
7. Richards, S.J., "Climatic control by building design". S.A. Architectural record, Vol. 44, No. 1, Jan., 1959.

Such treatment is not, however, always possible due to the scarcity of complete records of the required elements. In particular, data regarding long wave radiation exchange is almost non-existent, and the radiative sky and ground temperatures, invariably, have to be estimated.

The records obtained from the Sudan Meteorological Service, for Khartoum, were:

1. Hourly screen air temperatures for the years 1961-63.

2. Daily totals of solar radiation, direct and diffuse, falling on a horizontal surface, and daily sunshine hours, for the years 1961-63.

3. Mean hourly values of wind speed, for each month of the year, over the period 1949-53.

4. Mean hourly values of relative humidity, for each month of the year, over the period 1949-53.

The method adopted for the selection of design data is to choose, by inspection of the air temperature and sunshine records, a typically hot and clear day, for which hourly values of solar radiation can be computed. The design wind speed is chosen from the mean hourly values for the month. Radiative sky and ground temperatures are estimated from other sources (see 3.1.6.). Design values of each element, thus determined, are combined to form a hypothetical design day.

3.1.4. Air Temperature

Inspection of the Isopleths of Khartoum temperatures,* Fig. 1, shows that the highest daily maximum values of air temperature are reached in late April, May, and early June. Records of hourly air temperatures for Khartoum, for the years 1961-63, show that temperatures up to 112°F are reached in the early afternoon.

The highest daily totals of solar radiation occur at the same time of the year, though days with the highest totals of solar radiation do not necessarily have the highest air temperature maxima. Daily totals of 2210 Btu/ft^2 (600 cal/cm^2) are not uncommon (about 30 days a year), with the highest daily total recorded for the three years at about 2390 Btu/ft^2.

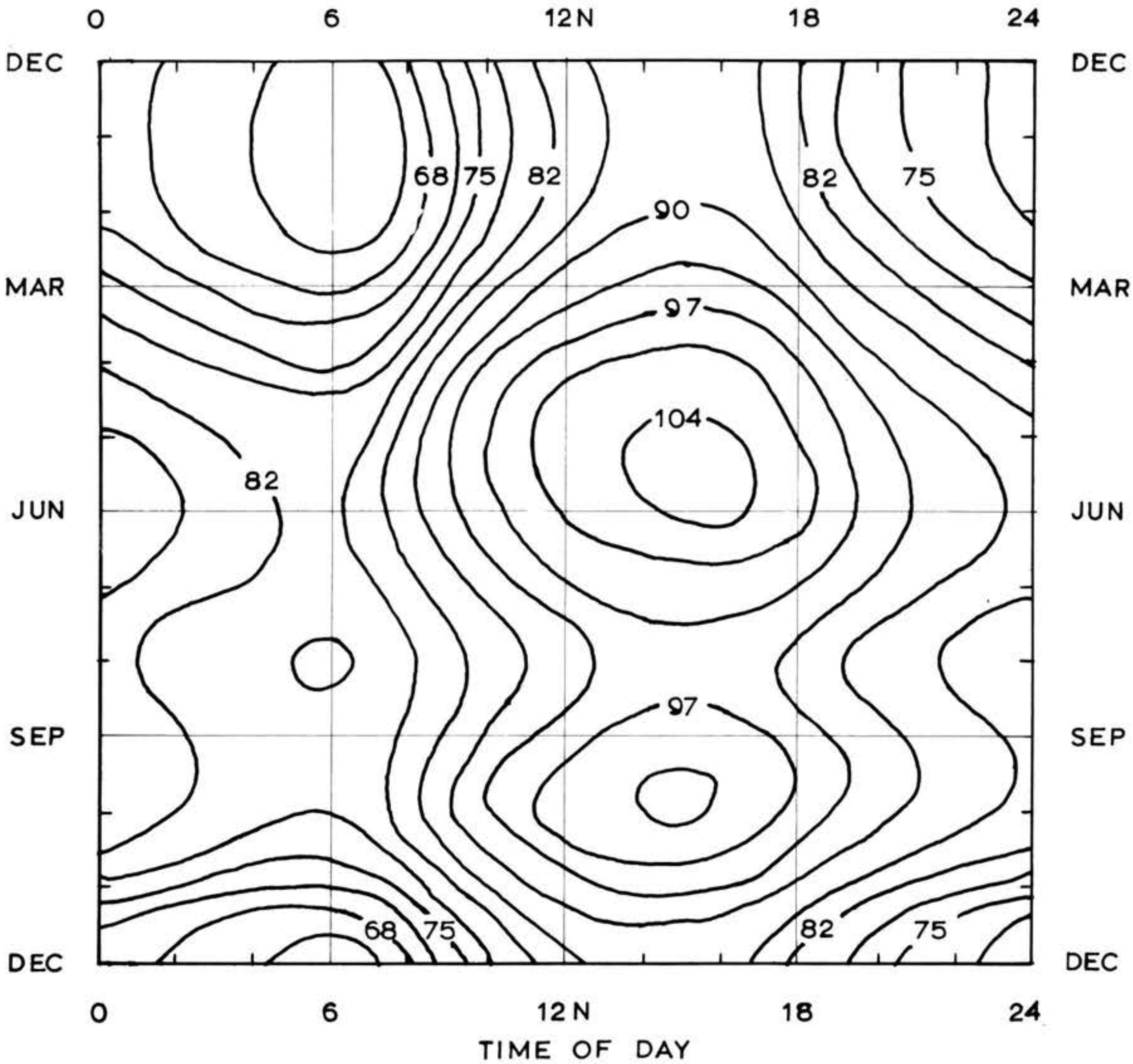
Spells of warm days with similar high temperatures and solar radiation totals sometimes occur. One such day (24th May 1962) seemed typical enough to be representative of problem summer conditions.

Maximum air temperature for the day, occurring at 2.00p.m. local time, was 111°F, the minimum 82°F occurring at 6.00 a.m. local time. The total solar radiation for the day (falling on a horizontal surface) was 2215 Btu/ft^2. 12.0 hours of sunshine were recorded.

The actual recorded hourly values of air temperature are given in Table 3.2 . Since in practice these values would be continuous with those of the days preceding and following the period chosen, a slight adjustment was necessary to make the values continuous around midnight and to smooth the 24-hour curve. The adopted design values are included in Table 3.2 .

---

* From Bhalotra, loc. cit., see footnote 3.4.



ISOPLETHS OF KHARTOUM TEMPERATURES (1945-52), DEG. F

FIG. 1.

Table 3.2

Actual and design air temperatures for the design day.

Hour of Day (local time)	1	2	3	4	5	6	7	8	9	10	11	12
Recorded temperature in °F.	87	85	86	84	83	82	83	91	97	101	105	108
Design temperature in °F.	87	85	84	83	82	82	83	91	97	101	105	108

Hour of Day (local time)	13	14	15	16	17	18	19	20	21	22	23	24
Recorded temperature in °F.	109	111	111	109	109	106	105	96	94	94	94	93
Design temperature in °F.	109	111	111	109	108	106	103	100	97	94	91	89

3.1.5. Solar Radiation

3.1.5.1. General

Since only records of daily totals of solar radiation, falling on a horizontal surface, were available for Khartoum, it was necessary to compute the hourly rates of radiation, falling on surfaces of different orientations.

The calculations are made for the 'Design Day', the 24th of May. The surfaces chosen are a horizontal surface, and vertical surfaces facing east, west north and south. This choice stems initially from the tradition for flat roofed buildings in dry climates, and the traditional siting of buildings, along cardinal axis, to catch the predominantly north-south wind* for cross-ventilation.

---

* loc. cit. see footnote 3.4.

On the earth's surface two components of radiation can be distinguished:

(1) that portion which escapes atmospheric depletion, known as direct radiation

(2) the portion received from the atmosphere, known as diffuse radiation, and which for some surfaces may include a contribution due to the reflection of direct and sky radiation from the ground and other objects.

The intensity of the direct component received by a plane, of a particular orientation, on the earth's surface depends on:[8]

(1) the intensity of radiation at normal incidence outside the atmosphere.

(2) the depletion of the radiation due to the atmosphere.

(3) the position of the sun with respect to the plane, as determined by the latitude, time of day and time of year, and expressed by the solar altitude and azimuth angles.

The intensity of solar radiation at normal incidence, at the outer limit of the atmosphere, and at mean solar distance (of the earth from the sun), is known as the 'Solar Constant'. The most accepted value of this Constant is 419.40 Btu/ft^2.hr.*

A surface perpendicular to the sun's rays at the earth's surface would, however, receive much less solar energy as a large portion of the radiation is scattered in passing through the atmosphere and absorbed by some of the atmospheric constituents, notably water vapour, ozone and carbon dioxide.

The starting point in solar radiation calculations is the determination

---

8. Jordan, R.C., and Threlkeld, J.L., "Availability and utilization of Solar energy". Report No. 1502, ASHVE Transactions, Vol.60, 1954.

* Ibid.

of the intensity of the direct beam, at the earth's surface, for a standard atmosphere. Values of the direct radiation for a plane, perpendicular to the sun's rays, on the earth's surface, as a function of solar altitude, were calculated by Parry Moon[9] for sea level and a standard clear atmosphere (of 20 m.m. precipitable water vapour, 300 dust particles per cubic cm. and 2.8 m.m. Hg partial ozone pressure). Moon's values are given in Table 3.3. .

Table 3.3.
Moon's values of direct normal radiation at the earth's surface as a function of solar altitude.

Solar alt. in Deg.	5	10	15	20	25	30	35	40	45	50	60	70	80	90
Moon's value Btu/hr.sq.ft.	67	123	166	197	218	235	248	258	266	273	283	289	292	294

Any other clear atmosphere can be defined with respect to Moon's Standard atmosphere. The ratio of the observed direct, normal intensity of radiation under such an atmosphere, to Moon's value, called the clearness number or ratio, gives an indication as to whether that atmosphere is clearer, or less clear, than the one proposed by Moon.

There is, however, no evidence that a clear tropical atmosphere differs appreciably from that defined by Moon. Rao and Seshadri[10] defined a tropical atmosphere as one differing from Moon's in having 15 m.m. precipitable water vapour (Moon, 20 m.m.) and 2.5 m.m. partial ozone pressure (Moon 2.8 m.m.). This choice is reported to have initially been arbitrary. The values of

9. Moon, Parry, "Proposed standard radiation curves for engineering use". Journal of Franklin Institute, Vol. 230, Nov. 1940.

10. Rao, K.R., and Seshadri, T.N., "Solar insolation curves". Indian Journal of Meteorology and Geophysics, Vol. 12, No.2, April 1961.

direct normal radiation computed by Rao and Seshadri are, however, only slightly different from Moon's.

For the purpose of computing direct solar radiation for Khartoum an atmosphere with a clearness number of unity is assumed and the calculations are based on Moon's values of direct, normal radiation.

3.1.5.2. The direct component

The intensity of the direct component of radiation falling on any plane, at the earth's surface, is given by:[11,12]

$$I_D = I_N \cos\theta \quad \ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

where,

I_D = intensity of the direct beam, in Btu/hr.sq.ft.

I_N = intensity of direct, normal radiation, for a particular solar altitude, Btu/hr.sq.ft.

θ = the angle of incidence for the plane, between the direction of the sun's rays and the normal to the plane.

For a horizontal surface:

$$\cos\theta = \sin\beta \quad \ldots\ldots\ldots\ldots \quad (3.2)$$

where β is the solar altitude angle, measured in a vertical plane between the sun's rays and the horizontal.

---

11. Threlkeld, J.L., and Jordan, R.C., "Solar radiation during cloudless days". Report No.1533, ASHVE Transactions, Vol. 61, 1955.

12. also see, Threlkeld, J.L., "Thermal environmental engineering". Prentice-Hall International, Englewood, N.J., 1962.

The direct component of solar radiation falling on a horizontal surface (I_{DH}) is therefore:

$$I_{DH} = I_N \sin \beta \text{ .... } (3.3)$$

For a vertical surface:

$$\cos \theta = \cos \beta \cos \gamma \text{ ..... } (3.4)$$

where γ is the surface solar zimuth angle, measured in a horizontal plane between the normal to the surface and the horizontal projection of the sun's rays.

The direct component falling on a vertical surface (I_{DV}) is, therefore:

$$I_{DV} = I_N \cos \beta \cos \gamma \text{ ..... } (3.5)$$

These solar angles vary continuously from sunrise to sunset, but are symmetrical with respect to the solar noon.* To make use of this symmetry, in solar radiation calculations, it is beneficial to use solar time, measured by the apparent diurnal motion of the sun, instead of civil time.

The difference between local civil time, for a particular longitude, and local solar time is called the equation of time.** Thus,

$$\text{L.C.T.} = \text{L.S.T.} - \text{Equation of time ..... } (3.6)$$

The equation of time varies from day to day and is due to the non-uniformity of the earth's movement around the sun. For the 'Design Day', the 24th of May, the equation of time is approximately 3.5 minutes and can, therefore, be ignored. Local civil time in Sudan is reckoned for the Khartoum meridian.

---

* Solar noon is the time when solar altitude is at its diurnal maximum.

** Ibid., see footnote 3.12.

Using a solar chart[13] for latitude 16^0N (Khartoum is at latitude 15^o:36') the altitude angle β and the surface solar azimuth γ, for surfaces east, west, north and south, are determined for each hour of the day. These angles together with values of direct normal radiation for the proper altitude angle, read from Table 3.3. , are given in Table 3.4. .

Table 3.4.

Altitude angle β , surface solar azimuth angle γ and the direct normal radiation I_N

Time	β in Deg.	I_N Btu/hr. sq.ft.	Surface Solar Azimuth angle γ in Deg.			
			East	West	North	South
6 a.m.	6	78	19	161	71	109
7 a.m.	21	201	16	164	74	106
8 a.m.	34	246	14	166	76	104
9 a.m.	47	269	12	168	78	102
10 a.m.	60	283	12	168	78	102
11 a.m.	74	290	17	163	73	107
Noon	85	293	90	90	0	180
1 p.m.	74	290	163	17	73	107
2 p.m.	60	283	168	12	78	102
3 p.m.	47	269	168	12	78	102
4 p.m.	34	246	166	14	76	104
5 p.m.	21	201	164	16	74	106
6 p.m.	6	78	161	19	71	109

When the surface solar azimuth angle γ is greater than 90^o, the surface does not 'see' the sun and is, therefore, in shade. This is the case for the east facing surface after noon, the west facing surface before noon, and the south facing surface all day.

13. Hopes, "Solar charts for daylight planning" H.M.S.O. London, 1961.

The hourly values of direct solar radiation falling on a horizontal surface, as given by equation (3.3), and on vertical surfaces facing east, west, north and south, as given by equation (3.5), are presented in Table 3.6 .

3.1.5.3. The diffuse component

Parmelee[14] proposed the following relationship between the direct and diffuse radiation falling on a horizontal surface on a clear day:

$$I_{dH} = X - Y(I_{DH}) \text{ .... } (3.7.)$$

where,

I_{dH} = diffuse radiation falling on the surface, in Btu/hr. sq. ft.

I_{DH} = the direct component falling on the surface, in Btu/hr. sq. ft.

X,Y = constants, depending on the solar altitude angle.

So that if the direct component of radiation is known the diffuse component can be estimated. Values of the constants X and Y, as a function of the solar altitude angle β , are given in Table 3.5. .

Table 3.5.

The constants X and Y in equation (3.7) as a function of the solar altitude angle.

β in Deg.	X	Y
10	20	0.295
20	42.7	0.314
30	70.4	0.360
40	90.1	0.362
50	121.4	0.424
60	153.6	0.492
70	175	0.520
80	192	0.545
90	198	0.560

14. Parmelee, G.V., "Irradiation of vertical and horizontal surfaces by diffuse solar radiation from cloudless skies". Report No. 1510, ASHVE Transactions, Vol. 60, 1954.

Equation (3.7) was used to estimate the diffuse component of radiation incident on a horizontal surface. Values of the constants X and Y for the relevant solar altitude angles (see Table 3.4.), were found by interpolation from Table 3.5. .

Parmelee* also produced curves giving the diffuse component of solar radiation for vertical surfaces as a function of solar altitude and surface solar azimuth angles and the clearness of the atmosphere. The curves for a clearness number of unity were used to estimate the diffuse radiation incident on surfaces facing east, west, north and south, using the solar altitude and surface solar azimuth angles given in Table 3.4. .

Since the data Parmelee used in preparing the abovementioned curves were collected under conditions where the immediate surroundings had an average reflectance of 10% and approximately 12°-15° horizon obstruction, the following corrections suggested by Parmelee were applied to the estimated values:

1 - A correction (I_g) for ground reflectance, to be added to the estimated value, given by the expression:

$$I_g = 0.5\,(I_{DH} + I_{dH})\,(r - 0.1) \quad \ldots\ldots \quad (3.8)$$

where

I_{DH}, I_{dH} = direct and diffuse radiation incident on a horizontal surface, for a particular solar altitude angle, in Btu/hr. sq. ft.

r = ground reflectance expressed as a decimal.

A value of $r = 0.3$ was used for the correction. This value seemed appropriate and is commonly quoted in the literature.

---

* Ibid, see footnote 3.14.

2 - A correction (I_h) for a clear horizon given by:

$$I_h = 0.15\ I_{dV(\gamma > 90)} \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

where

$I_{dV(\gamma > 90)}$ = diffuse component of radiation for a vertical surface that does not 'see' the sun, in Btu/hr. sq. ft.

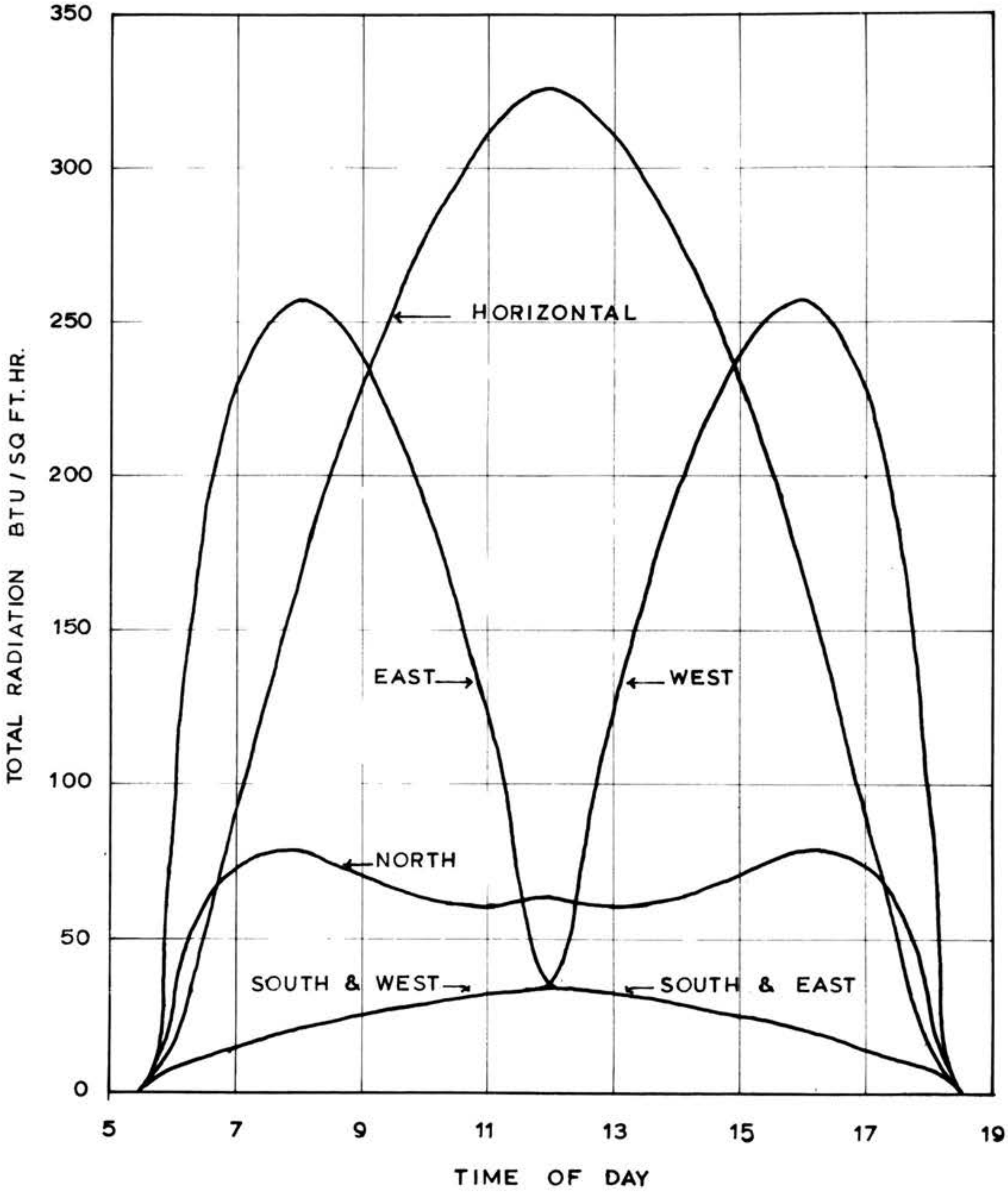
In this case the values of the diffuse component for the south facing surface were used in equation (3.9)

The hourly values of diffuse radiation falling on a horizontal surface and on vertical surfaces facing east, west, north and south are given in Table 3.6. . The Table also gives the hourly values of total radiation, direct and diffuse, for each of the surfaces considered. These latter values are also plot in Fig. 2.

Table 3.6

Computed hourly values of direct (I_D), diffuse (I_d), and total (I_T) radiation falling on a horizontal surface and vertical surfaces facing east, west, north and south at latitude 16°N on the 24th of May

TIME	Horizontal			East			West			North			South		
	I_D	I_d	I_T	I_D	I_d	I_T	I_D	I_d	I_T	I_D	I_d	I_T	I_D	I_d	I_T
6 a.m.	8	8	16	73	18	91	0	4	4	25	7	32	0	8	8
7 a.m.	72	20	92	180	47	227	0	13	13	52	21	73	0	14	14
8a.m.	137	29	166	198	59	257	0	21	21	48	30	78	0	21	21
9 a.m.	197	31	228	180	58	238	0	25	25	38	32	70	0	25	25
10 a.m.	245	33	278	138	53	191	0	29	29	29	34	63	0	29	29
11 a.m.	278	34	312	76	45	121	0	32	32	25	35	60	0	32	32
Noon	292	34	326	0	34	34	0	34	34	26	37	63	0	34	34
1 p.m.	278	34	312	0	32	32	76	45	121	25	35	60	0	32	32
2 p.m.	245	33	278	0	29	29	138	53	191	29	34	63	0	29	29
3 p.m.	197	31	228	0	25	25	180	58	238	38	32	70	0	25	25
4 p.m.	137	29	166	0	21	21	198	59	257	48	30	78	0	21	21
5 p.m.	72	20	92	0	13	13	180	47	227	52	21	73	0	14	14
6 p.m.	8	8	16	0	4	4	73	18	91	25	7	32	0	8	8



TOTAL SOLAR RADIATION FALLING ON A HORIZONTAL SURFACE AND VERTICAL SURFACES FACING EAST WEST NORTH & SOUTH AT KHARTOUM COMPUTED FOR SUMMER DESIGN DAY

FIG. 2.

3.1.5.4. Computed and recorded values

A comparison of the computed and recorded values is possible only for total daily radiation falling on a horizontal surface since no values for vertical surfaces are recorded. The computed daily total, for a horizontal surface, amounts to 2507 Btu/sq.ft. The total recorded for the day is 2215 Btu/sq.ft. The higher computed values are partly explained by the fact that the computed values assume approximately 13 hours of sunshine, according to the solar charts used,* where the recorded values were for 12 hours sunshine. The computed value, however, reduces to 2442 Btu/sq.ft. when fitted into the function generator scheme (see section 2.2.2.4), which is approximately 50 Btu/sq.ft. more than the highest value recorded for the three years of record (1961-63) considered. This difference (2%) is smaller than the possible percentage error ($2\frac{1}{2}$%) involved in generating the function in the analogue set-up.

3.1.6. Radiant environmental temperature

3.1.6.1. General

It has been shown, in section 1.4.2, that to account for long wave radiation exchange between a horizontal surface and its environment a knowledge of the temperature at which the sky radiates is necessary. Also that the temperature at which the environment radiates to a vertical surface can be taken as the mean of sky and ground temperatures (see equation 1.27). A knowledge of the temperature of the ground in the vicinity of the building surface is, therefore, also desirable

These temperatures can, however, only be estimated; a task which is made more difficult since a 24-hour curve is required. It should however

---

* See footnote 3.13.

be noted that a thermal circuit is less sensitive to small variations in radiative temperatures than to variations in air temperature due to the higher surface resistances for radiative heat transfer.

3.1.6.2. Sky temperature

If the sky is assumed to radiate at a temperature T_{sky}, then this temperature can be estimated from a knowledge of the ground level screen air temperature and vapour pressure, using Brunt's formula, given by equation (1.16) as:

$$R_{sky} = R_{bo}(a + b\sqrt{p})$$

where,

R_{sky} = radiation emitted by the sky, Btu/ft.^{2}hr.

R_{bo} = radiation emitted by a black body at screen air temperature, Btu/ft.^{2}hr.

a = 0.55, a constant.

b = 0.056, a constant.

p = vapour pressure in millibars.

From the above equation:

$$\sigma T_{sky}^4 = \sigma T_o^4 \ (a + b\sqrt{p})$$

where, σ = Stephan Boltzmann constant.

or

$$T_{sky} = T_o \ (0.55 + 0.056\sqrt{p})^{\frac{1}{4}} \quad \text{..........} \quad (3.10)$$

where

T_{sky} and T_o = absolute sky and screen air temperatures in R°.

The actual values of the radiative sky temperatures will, therefore, depend on the choice of vapour pressure values. Records of these could not be obtained for Khartoum.

Page* using Brunt's formula to estimate radiative sky temperatures for a hot dry region, assumed a value of p = 20 mb, admittedly a higher value than expected in such regions.

Buchberg,[15] using Brunt's formula to compute radiative sky temperatures for the Frenso area, U.S.A., for a clear summer day, based the calculations on actual records of vapour pressure. Working backwards from the sky and air temperatures presented by Buchberg, it is found that the vapour pressure in Frenso varied between approximately 9 mb, at the time of minimum air temperature just before sunrise, and 11.5 mb, at the time of maximum air temperature in the early afternonn.

These values of vapour pressure existed for a hot clear day, similar to the Khartoum 'design day', with an air temperature maximum of 105°F and a noon intensity of solar radiation, falling on a horizontal surface, of 302 Btu/ft^2.hr.

Using the same values to compute the sky temperatures at the times of maximum and minimum air temperature in Khartoum (using equation 3.10) gives a maximum sky temperature of 70°F (6°F higher than Frenso's, which is the same as the difference in air temperature maxima for Khartoum and Frenso), and a minimum sky temperature of 40°F (11°F higher than Frenso's, which is the same as the difference in air temperature minima for Khartoum and Frenso).

---

* loc.cit. see footnote (1.5).

15. Buchberg, H., and Naruishi, J., "Low income housing demonstration thermal analysis", Report 65-11, U.C.L.A., March, 1965.

It was, therefore, tempting to follow Buchberg's values, since these are based on actual vapour pressure records. A curve similar to Buchberg's was assumed to exist for Khartoum, exhibiting a maximum sky temperature of $70^{o}F$ and a minimum of $40^{o}F$.

The curve of Buchberg's values of sky temperature together with the curve adopted for Khartoum are given in Fig. 3. The values for Khartoum are listed in Table 3.7 .

Table 3.7

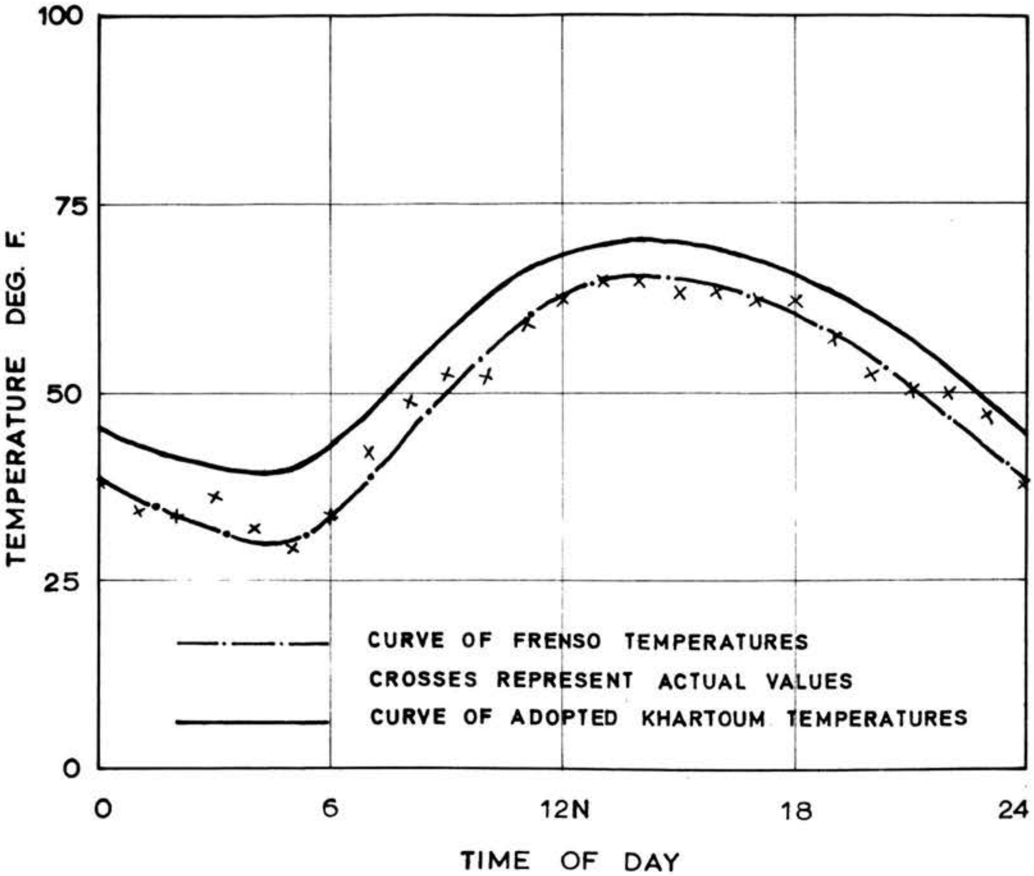
Estimated Sky Temperature (T_{sky}) for Khartoum.

Hour of Day (local time)	1	2	3	4	5	6	7	8	9	10	11	12
Sky Temperature ^{o}F	43	42	41	40	40	44	50	54	58	64	66	68
Hour of Day (local time)	13	14	15	16	17	18	19	20	21	22	23	24
Sky Temperature ^{o}F	69	70	70	69	68	66	63	60	57	52	48	45

3.1.6.3. Ground temperature

The temperatures attained by the ground were estimated by solving the thermal circuit representing the ground as a semi-infinite slab subjected to the diurnal variation of air temperature, (Table 3.2), the solar radiation falling on a horizontal surface (Table 3.6), and long wave radiation exchange with the sky.

The thermal circuit is discussed, and the analogue solution given, in Appendix 2 . The estimated ground temperatures are listed in Table 3.8 .



ESTIMATED SKY TEMPERATURE CURVE FOR KHARTOUM

FIG. 3.

The table also gives the hourly means of sky and ground temperatures, which are the required environmental temperatures for radiation exchange between a vertical surface and its environment, as given by equation (1.27).

Table 3.8

Estimated Ground temperatures (T_{gro}) and the hourly means of ground and sky temperatures; for Khartoum.

Hour of Day (local time)	1	2	3	4	5	6	7	8	9	10	11	12
Ground temperature ^oF	84	82	80	78	75	79	83	98	106	114	123	129
$\frac{T_{sky} + T_{gro}}{2}$, ^oF	64	62	61	59	58	62	69	76	82	89	95	99

Hour of Day (local time)	13	14	15	16	17	18	19	20	21	22	23	24
Ground temperature ^oF	134	137	133	126	119	112	105	100	95	92	89	86
$\frac{T_{sky} + T_{gro}}{2}$, ^oF	102	104	102	98	94	89	84	80	76	72	69	66

3.1.7. Design Wind Speed

The mean hourly values of wind speed, at Khartoum, for the month of May, over the period 1949-53, are given in Table 3.9 .

Table 3.9

Mean hourly wind speeds for May (1949-53), at Khartoum.

Hour of Day	1	2	3	4	5	6	7	8	9	10	11	12
Wind speed M.P.H.	8.1	8.5	8.2	8.0	8.4	7.8	7.9	9.4	11.6	11.7	11.0	9.6

Hour of Day	13	14	15	16	17	18	19	20	21	22	23	24
Wind speed M.P.H.	8.8	8.2	8.0	7.7	7.5	7.1	6.5	6.3	6.8	7.2	8.0	8.7

It will be seen that the lowest speeds occur a few hours before midnight while the highest occur just before noon. The most frequent values lie between 8 and 9 M.P.H., with the mean of all values at 8.4 M.P.H., so that a value of this order can be assumed to represent the average free wind conditions.

The problem is to choose a design wind speed for the purpose of determining an appropriate convection coefficient, at the outside surface of a building element. This coefficient (h_{co}) is assumed constant for the 24-hour cycle.

Wind speed, however, varies not only with time, but also with height and degree of obstruction, so that the free wind speed would have to be modified to approximate conditions in built-up areas. Page* adopted a scheme whereby the free wind speed is modified for built-up areas to be:

---

* loc. cit. see footnote (1.5).

(i) one-third of the free wind speed when considering the first few storeys of buildings in central parts of towns.

(ii) two-thirds of the free wind speed for buildings in suburban areas, and for the middle stories of buildings in the central parts of towns.

(iii) equal to the free wind speed where there is no wind screen, and for the upper floors of buildings.

If a value of two-thirds of the free wind speed is assumed for Khartoum because of its low building density, then considering a free wind speed of 8.4 M.P.H., a design value, for built-up areas, would be in the order of 5.5 M.P.H.

It is not easy, however, to prove that the above scheme, for the modification of free wind speed, is a rigorous formulation. Nevertheless, the use of a design wind speed of 5.5 M.P.H. leads, as will be shown in the next chapter (see section 4.4.1), to a heat transfer convection coefficient (h_{co}) of a magnitude comparable to similar coefficients quoted in the literature.

3.2. THERMAL COMFORT AND DISCOMFORT

3.2.1. Thermal stress

Human beings expend energy they obtain from the oxidation of food on work and on the maintenance of a constant body temperature. When body heat production is in excess of body energy requirements the body loses heat to the surroundings. Body temperature is controlled by physiological mechanisms which regulate the rate of heat production within the body and the rate of heat loss.

Heat loss takes place by radiation, convection and evaporation, the rate of loss depending on the temperature of air, the mean temperature of the surroundings and the movement and humidity of air. Under neutral thermal conditions the regulation of heat production and loss is accomplished by vasomotor control - control of blood flow in the skin and subcutaneous tissue - and insensible perspiration[16].

Two emergency mechanisms, however, exist. Chemical regulation - the increase of metabolic heat production by shivering, and evaporative regulation - through the loss of heat by evaporation of sweat. The former mechanism is associated with body heat deficit, the latter with body heat excess.

When the thermal balance between the body and its environment is upset, through inadequate heat loss or by heat gain from the environment a condition of thermal stress is said to exist. This is a condition frequently met with in warm climates, and have been shown[17] to lead, not only to subjective

---

16. MacPherson, R.K., "Physiological aspects of thermal comfort". Architectural Science Review, Vol. 8, No. 4, 1965.
17. Givoni, B, and Rim, Y., "Effect of the thermal environment and psychological factors upon subjects' response and performance of mental work". Ergonomics, Vol. 5, No. 1, 1962.

sensations of discomfort, but also to a reduction in mental and physical efficiency, and in extreme cases, to constitute a hazard to health.

Heat balance, though necessary, is not on its own sufficient to produce a feeling of thermal comfort. The arrangement of environmental factors, e.g. thermal gradients, and qualitative elements, such as freshness and stimulation, may play an important part in the response of an individual to the environment. Environmental comfort must also be associated with agreeable sonic and luminar conditions.

3.2.2. Effect of different environmental factors on warmth sensation.

No attempt will be made to discuss the numerous indices available for the measurement of the thermal environment. These are adequately described in the literature, and reference may be made, for example, to Bedford[18] or Givoni[19]. It is, however, useful to consider the contribution of the different environmental factors to warmth sensation.

The factors affecting heat exchange between the body and the environment are the temperature and movement of air, the radiant temperatures of surroundings, and the humidity of air. Air movement affects the heat exchange in two ways: by affecting the convective heat exchange, and the cooling efficiency of sweat evaporation*. The effect of intensified air motion on these two factors, when the air temperature is below that of the skin, is in the same direction, namely towards increased body cooling.

---

18. Bedford, T., "Basic principles of ventilation and heating". Lewis and Co., London, 1966.
19. Givoni, B., "Estimation of the effect of climate on man - Development of a new thermal index". B.R.S. Haifa, 1963.

* Ibid.

When, however, the air temperature exceeds that of the skin, intensified air motion produces higher convective heat gains and, at the same time, may produce a cooling effect by increasing the cooling effect of sweat evaporation. In such circumstances the net effect of air movement may be beneficial or harmful depending on conditions of humidity, metabolic rate and clothing.

Radiant exchange between the body and surroundings may have a cooling or warming effect according to whether the mean radiant temperature of surroundings is lower or higher than skin temperature. Skin radiates and absorbs long wave radiation almost like a black body.*

Bedford* reported that where the mean temperature of the surroundings is not higher than 90°F, the effect of a given change in the radiant temperature is, in still air, practically the same as that of a similar change in air temperature. However, as wind velocity increases the effect of radiant temperature decreases relative to air temperature so that at a velocity of 100 ft per minute, a change of 10°F in the mean radiant temperature of the surroundings has the effect of a change of only 7.7°F in air temperature.

As sweat evaporation assumes importance at temperatures high enough to invoke sweating, the level of humidity is, as expected, relatively unimportant in thermally neutral environments. It has, however, been reported by Koch[20] that, although wide variations in humidity can take place in the comfort zone without affecting the feeling of thermal comfort, at 70°F dry bulb temperature, people may feel the air uncomfortably wet when the relative humidity exceeds 72%.

---

* loc.cit. see footnote 3.18.

20 Koch, W., "Humidity sensations in the thermal comfort range". Architectural Science Review, Vol. 6, No. 1, 1963.

At higher temperatures, when evaporative cooling assumes a major part in the heat exchange process, a high humidity leads to a reduction in the efficiency of sweat evaporation, thereby having a warming effect.

MacPherson,* also points out that since the body has an obligatory water loss from the respiratory system and the skin, low relative humidities, even in otherwise comfortable environments, may result in an unpleasant, and sometimes harmful, drying of nose, throat and skin.

3.2.3. Comfort standards for people in hot dry climates.

Some research in the thermal comfort standards of people in hot dry climates is available, from which temperatures favoured by people in such climates can be estimated. It should be noted, however, that any differences in preferences found between people in warm, and those in cold climates is due, only, to differences in clothing and acclimatisation.

Drysdale[21], from investigations in the hot arid regions of Australia found that subjects were most comfortable at 75°F dry bulb air temperature, but that they tolerated temperatures up to 84°F. Drysdale proposed that this latter temperature be considered the upper limit of the comfort zone.

MacPherson[22], conducted another investigation in Sydney, and reported that 80% of his subjects were comfortable at a dry bulb temperature of 73°F. However, using a 'comfortable' vote by 50% of the subjects as a criteria, the lower and higher limits of the comfort zone were found to be 66°F and 81°F,

---

* loc. cit. see footnote 3.16.

21. Drysdale, J.W., "Climate and the design of buildings". Commonwealth Experimental Building Station, Sydney, Technical study No. 35, 1950.
22. MacPherson, R.K., "Studies in the preferred thermal environment". Architectural Science Review, Vol. 6, No. 4. 1963.

dry bulb temperature, respectively. MacPherson, also, mentions an investigation by Rao in Calcutta, which indicated an optimum comfort temperature of 78.5°F, dry bulb.

van Straaten and van Deventer[23] report that the accepted optimum summer temperature, in the arid regions of South Africa, is 79°F, dry bulb temperature, with the upper limit of comfort at 82°F and the threshold of definite discomfort at 88°F. Webb[24] devised a method which consisted of finding points of change-over from warm daytime conditions to cool night-time conditions, inside tropical buildings, by obtaining a vote of 'comfortable' from a number of subjects, and then measuring the environment at the time of change-over. Applying the method in Roorkee, India, and Baghdad, Iraq, he found the environment judged comfortable in Roorkee to correspond to a globe and dry bulb temperatures of 86°F when the wet bulb temperature was 77°F. Comfortable conditions in Baghdad corresponded to dry bulb temperature of 95°F, a globe temperature of 97°F at a wet bulb temperature of 68°F.

In hot dry climates, with low humidities, if air movement can be assumed to be of a small order, inside typical buildings, and if the mean radiant temperature of the environment is not much higher than air temperature, then comfort standards can be expressed reasonably accurately in terms of dry bulb temperatures only. This is confirmed by the statistically significant correlation coefficient found, by Bedford,[*] between warmth sensation and dry bulb temperature, in the comfort zone.

From the above review, it seems safe to assume that optimum comfort

---

23. van Straaten, J.F., and van Deventer, E.N., "The functional aspects of building design in warm climates with particular reference to thermal and ventilation considerations". International Journal of Biometeorology, Vol. 8, No. 2, 1964.
24. Webb, C.G., "Thermal discomfort in a tropical environment". Research Series 22, B.R.S., London.
* loc.cit. see footnote 3.18.

conditions are realised at dry bulb temperatures 75 - 80°F, with the upper limit of the comfort range at 85°F, in conditions where humidity and air movement are small, and the mean radiant temperature of the environment not much higher than air temperature.

CHAPTER 4

EXPERIMENTAL PROGRAMME

4.1 Scope

The analogue studies were designed to explore the effect of the application and positioning of insulation, on the temperatures obtaining in buildings, under a variety of assumed conditions, in the hot dry climate of Khartoum.

The main consideration which shaped the programme was that all building response simulator models had to be built individually. The analogue set-up had no built-in R-C network. The time factor was, therefore, important and it was a matter of choice whether to build a few models of complete enclosures for testing, or to make the tests on a greater number and variety of single wall and roof elements, models of which are easier to build and handle.

Testing single elements requires certain assumptions to be made regarding the nature of the indoor boundary conditions; but the manipulation of variables is much easier than in the case of models of complete enclosures.

It was, therefore, decided to run two series of experiments. The first series dealt with single wall and roof elements; the second with a complete enclosure.

In the first series a number of walls and roofs of different types, including light, medium and heavy weight construction, were considered. Various orientation and indoor conditions were assumed. In each case insulation is applied to each element tested, in different amounts and positions, and a prediction of the surface temperatures of the element obtained at each stage.

In the second series tests were carried out on the model of an enclosure formed of selected wall and roof sections. A variety of indoor conditions were assumed, and in each case the effect of insulating wall and roof elements on the surface temperatures and the indoor air temperature was noted for different positions of the insulation.

A detailed description will now be given of the experimental set-up.

4.2. Single Elements

4.2.1. Wall and roof types

Three wall and three roof types were considered. The types were chosen to cover a cross-section of light, medium and heavy weight construction. The constructional details were drawn from common usage and the materials were chosen to be in the appropriate weight range considered.

Wall and roof models are designated by letters as follows:

Model T : Timber wall.
Model H : Hollow block wall.
Model B : Solid brick wall.
Model L : Timber roof.
Model O : Hollow trough roof.
Model C : Solid concrete roof.

4.2.1.1. Specification

All walls and roofs were assumed to have an area of 200 sq. ft. This gives total resistance and capacitance values within the optimum operational range of the analogue (see section 2.2.3.) and relates to the dimensions chosen for the enclosure making it possible to use some of the wall and roof simulator models in building the enclosure's simulator network.

Constructional details were as follows:

Model T : Timber wall, with 4 x 2 in. softwood studs at 18 in. centres, covered on the outside with $\frac{1}{2}$ in. softwood sheathing and finished with $\frac{3}{4}$ in. hardwood boarding. A paper vapour barrier is sandwiched between the sheathing and the face boards. On the inside $\frac{3}{4}$ in. softwood sheathing is faced with $\frac{1}{4}$ in. hardwood veneer. A plan and skeleton elevation of the wall are given in Fig. 1.

Model H : Hollow block wall made of hollow lightweight concrete blocks of 16 x 8 x 8 in. nominal size, and finished on both sides with $\frac{3}{4}$ in. plaster. Sectional drawings and dimensional details are given in Fig. 2.

Model B : Solid brick wall made of common brick, $13\frac{1}{2}$ in. thick, and finished on the inside with $\frac{3}{4}$ in. plaster. Sectional drawings of the wall are given in Fig. 3.

Model L : Timber roof, with 6 x 2 in. softwood joists at 18 in. centres, softwood tapered ferring averaging 1 in. in thickness, $\frac{3}{4}$ in. softwood boarding and felt roofing $\frac{3}{8}$ in. thick. The ceiling is $\frac{1}{2}$ in. softwood boarding under the joists. Sectional drawings are given in Fig. 4.

Model O : Hollow trough roof, made of reinforced concrete trough decks spanning the width of the roof, with 16 x 6 in. nominal section, topped with lightweight screed averaging 2 in. in thickness, and felt roofing $\frac{3}{8}$ in. thick.. The ceiling is $\frac{1}{2}$ in. asbestos-cement tiling under the trough decks. Sectional drawings are given in Fig. 5.

Model C : Solid concrete roof, with a 6 in. thick reinforced concrete slab topped with screed averaging 2 in. in thickness and $\frac{3}{8}$ in. felt roofing. The ceiling is finished with $\frac{3}{4}$ in. plaster. Sectional drawings are given in Fig. 6.

4.2.1.2. Physical properties

The physical properties required in this context are the thermal conductivity, density and specific heat of the materials entering into the construction of the wall and roof elements.

Some difficulty arises in choosing values of these properties as a wide variety of values can be found, for materials of similar description, in different sources of information. Also, since the thermal conductivity depends, among other things, on the moisture content of the material and the temperature at which the conductivity is measured, the choice is made more difficult because the moisture-temperature conditions are sometimes only partially described, and often do not cover the range of moisture-temperature conditions encountered in hot dry climates.

It was, therefore, sometimes necessary to interpolate, and match different sources of information, to obtain a comprehensive list of properties for some materials.

The main sources of information were: ASHRAE guide[1], IHVE guide[2], and Barned[3]. Minor sources of information, Ingersol et al[4] and Buchberg[*], were also consulted.

---

1. ASHRAE, "Guide and Data Book : Fundamentals and Equipment". 1963.
2. IHVE, "A guide to current practice", London, 1959.
3. Barned, J.R., "Thermal conductivities of building materials". Report No.2, Division of building research, C.S.I.R.O., Melbourne, 1954.
4. Ingersoll, L.R., Zobel, O.J., and Ingersoll, A.C., "Heat conduction". Thames and Hudson, London, 1955.

* loc. cit see footnote 2.15

A list of the properties chosen is given in Table 4.1.

Table 4.1.
Physical properties of materials
used in wall and roof constructions.

Material	Density lb/ft^3	Specific heat $Btu/lb.^{\circ}F$	Conductivity $Btu\ in/ft^2.hr.\ ^{\circ}F$
Softwood	32	0.4	0.8
Hardwood	45	0.42	1.1
Concrete (wall)	95	0.19	3.2
Plaster (cement-lime)	90	0.2	3.3
Common brick	120	0.2	5.0
Lightweight screed	60	0.2	3.0
Screed (sand-cement)	120	0.2	5.0
Concrete (roofs)	140	0.2	9.0
Asbestos-cement (tiles)	120	0.2	4.0
Air (wall air space)	0.08	0.24	Conductance 1.17
Air (roof air space)	"	"	Conductance 1.01
Paper vapour barrier	-	-	Conductance 16.7
Felt roofing ($\frac{3}{8}$ in.)	70	0.2	Conductance 3.0

Some of the wall and roof constructions enclose air spaces.

Heat is transmitted through an air space by radiation and convection as well as conduction. It is, therefore, not sufficient to use the conductive resistance of the air space on its own. The most convenient way of dealing with an air space is to determine its overall conductance.

The conductance of an air space is influenced by the width of the air space, the mean temperature, the temperature difference between the enclosing surfaces, the condition of the surfaces and, to a minor extent, the ratio of area to width of the air space.

Radiative transfer depends on the nature of the enclosing surfaces and their temperatures and is independant on the width of the air space. Convective transfer depends on the temperature difference between the enclosing surfaces and the lack of obstruction to air circulation. Conductive transfer is inversely proportional to the width of the air space.

For a vertical air space the convective transfer assumes greater importance as the width of the space is increased. Rowley and Algren[5] have shown that a point is reached when increasing the width of the air space has little effect on the overall conductance.

For practical purposes the conductance of an air space more than 1.5 inches wide may be assumed to depend only on the emissivities of the enclosing surfaces, the mean temperature and the temperature difference between the surfaces.

For usual building surfaces (emissivity = 0.9), a mean temperature of 90°F and 10°F temperature difference, the ASHRAE guide[*] gives a vertical air space conductance as 1.17, and a horizontal air space conductance (for heat flow downwards) as 1.01. These values are quoted in Table 4.1.

4.2.1.3 <u>Wall and roof simulator networks</u>

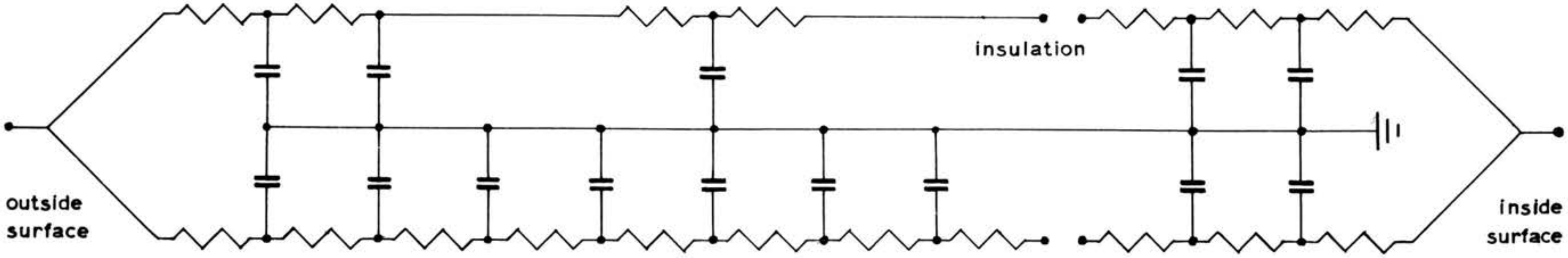
The resistances and capacitances of elements in the heat flow path are calculated using equations (2.18), (2.19), (2.22) and (2.23) given in section 2.1.4.

In the case of the brick wall (model B) and the solid concrete roof (model C) materials in the heat flow path are arranged in series. The simulator network consists of a single branch of T-sections with the resistances in series and

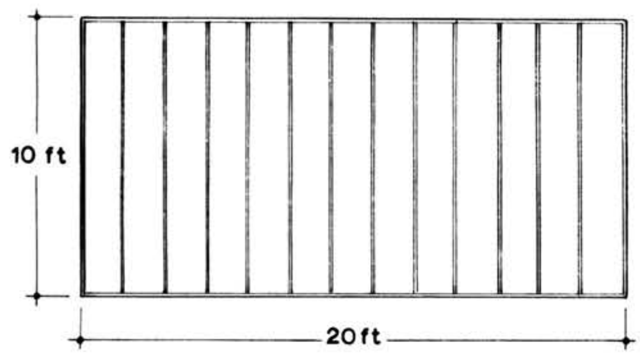
---

5. Rowely, F.B., and Algren, A.B., "Thermal resistance of air spaces". ASHVE Transactions, Vol. 35, 1929.

* loc. cit., see footnote 4.1.

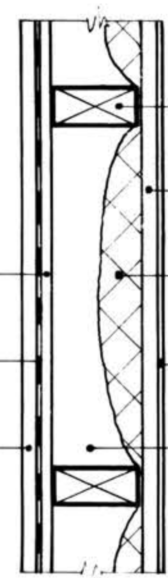


Electrical Model.



Skeleton Elevation.

wall area = 200 sq ft
stud area = 25 sq ft
air space area = 175 sq ft



Plan.

FIG. 1. TIMBER WALL — MODEL "T".

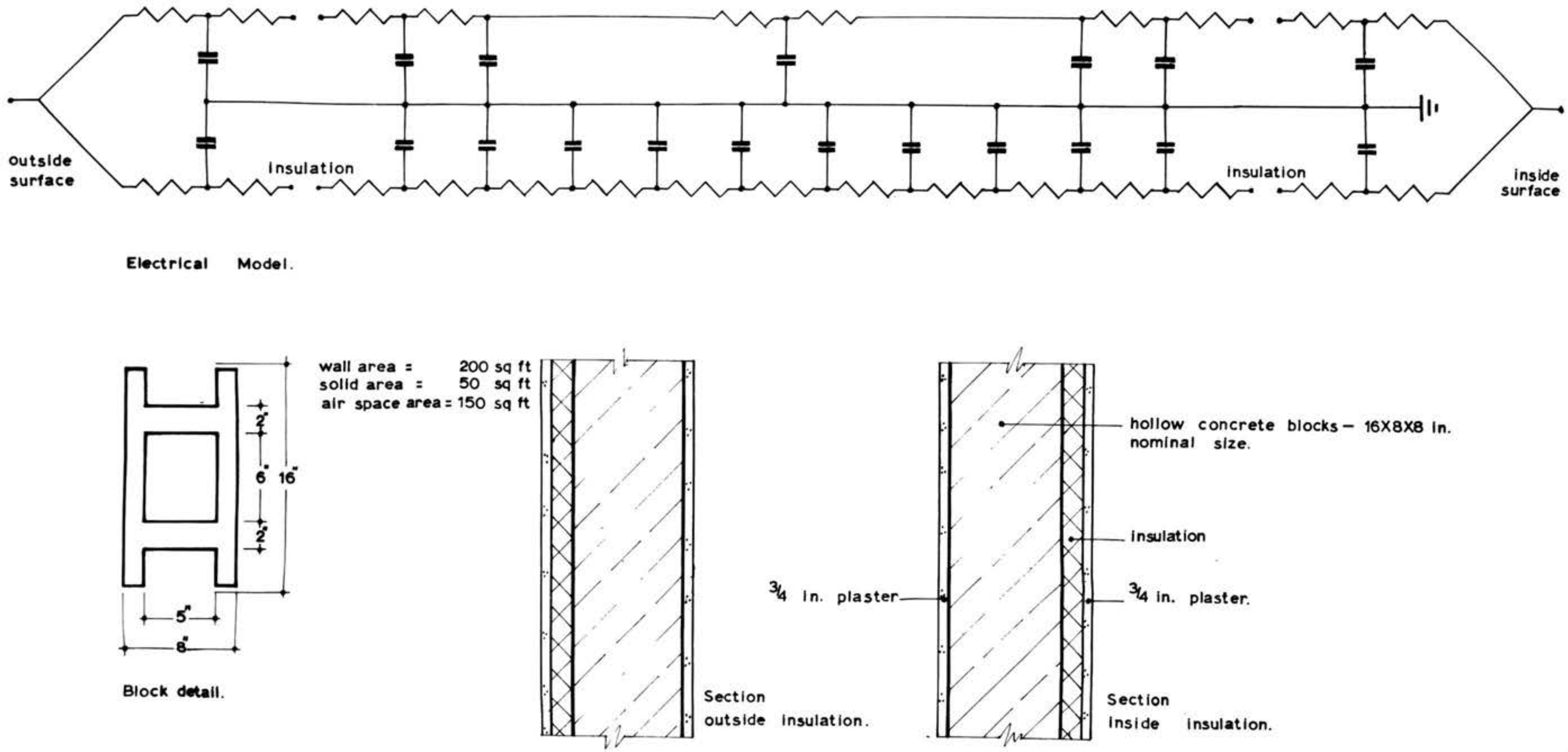
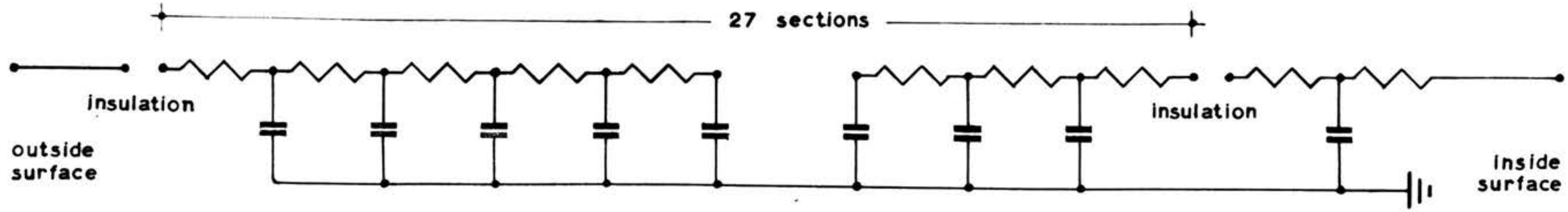
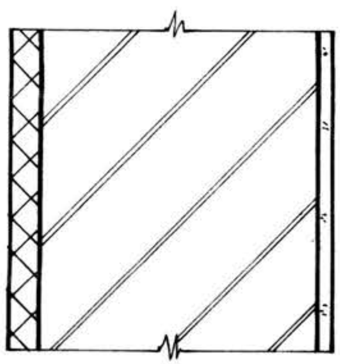


FIG. 2. HOLLOW BLOCK WALL – MODEL "H".

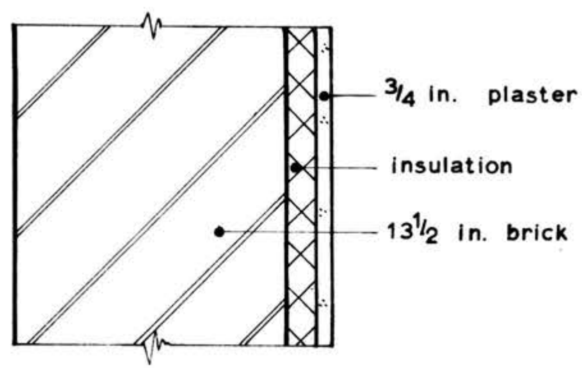


Electrical Model.



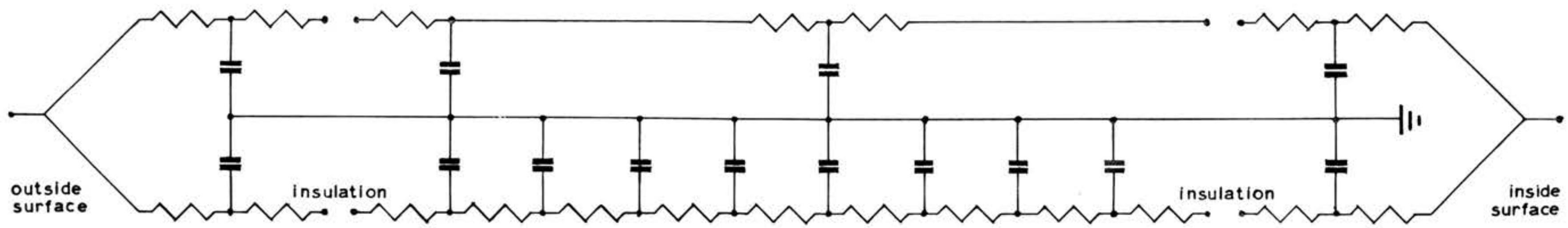
Section – outside insulation.

wall area = 200 sq ft.



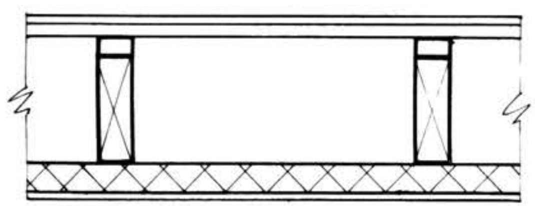
Section – inside insulation.

FIG. 3. BRICK WALL — MODEL "B".

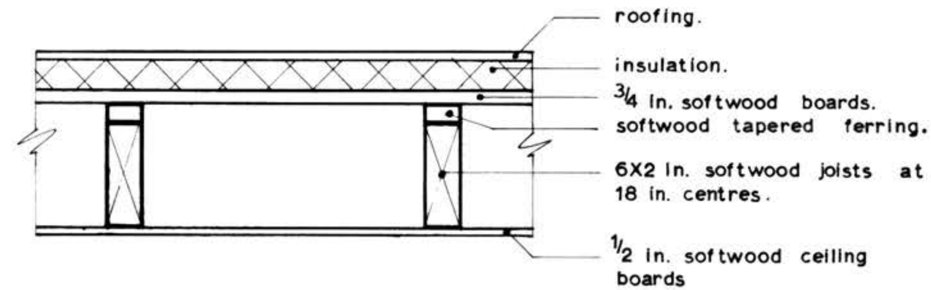


Electrical Model.

roof area = 200 sq ft
solid area = 25 sq ft
air space area = 175 sq ft



Section – insulation inside.



Section – insulation outside.

FIG. 4. TIMBER ROOF – MODEL "L".

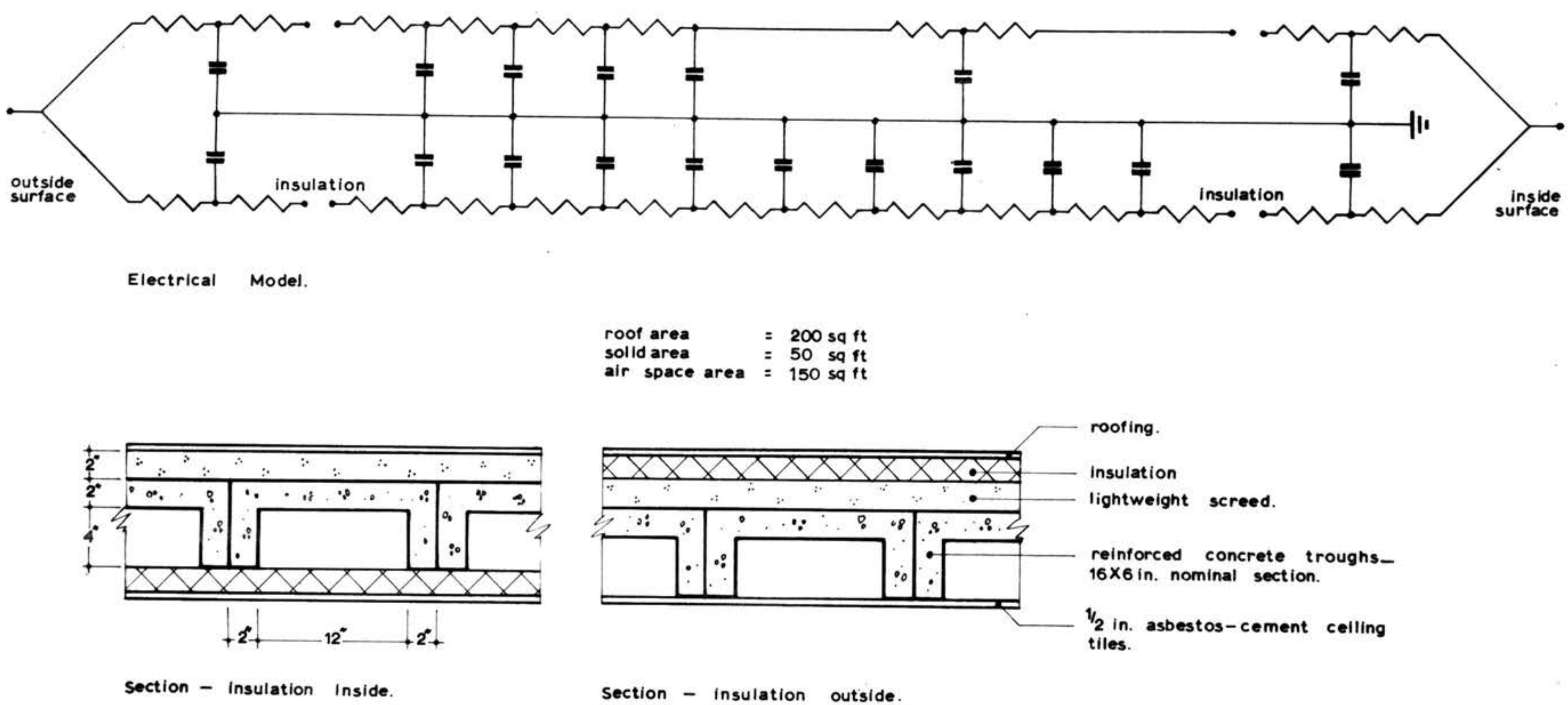


FIG. 5. HOLLOW TROUGH ROOF – MODEL "O".

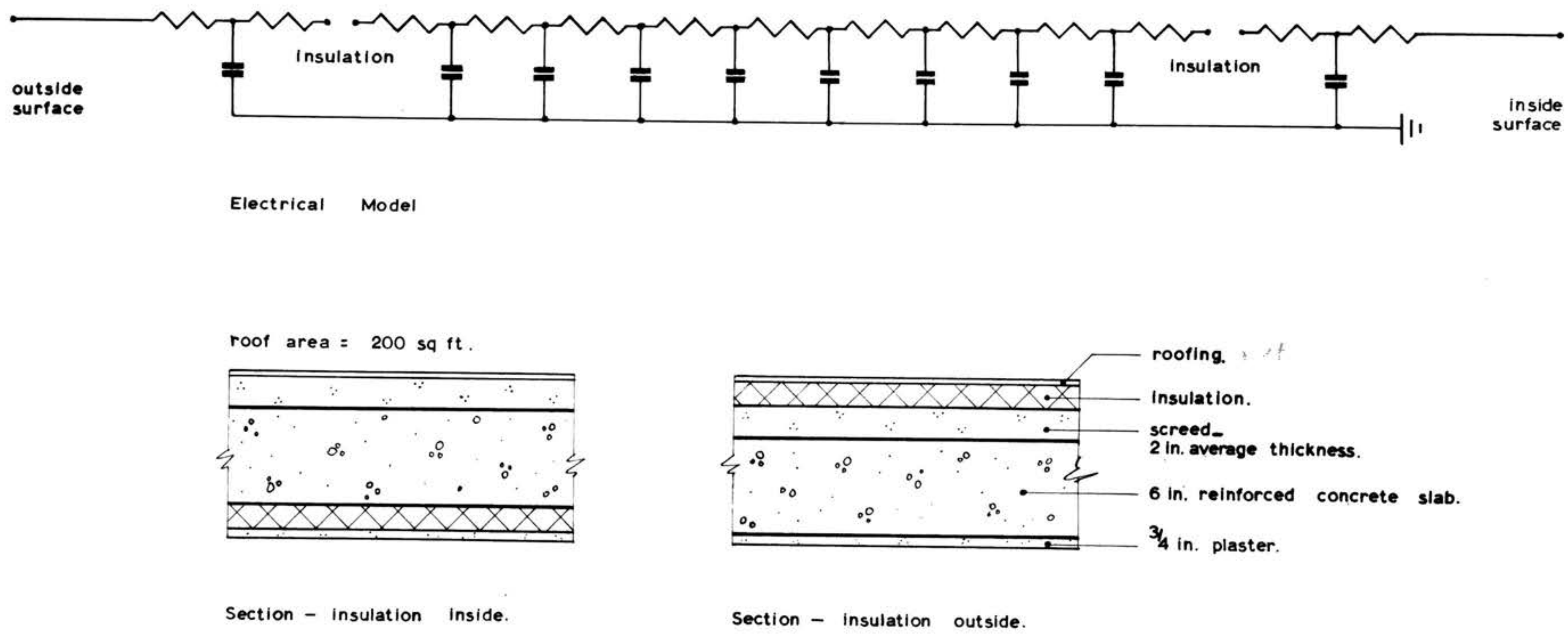


FIG. 6. SOLID CONCRETE ROOF — MODEL "C".

the capacitances in parallel (resistances add in series, capacitances add in parallel).

There is, however, the case of the other constructions where two conduction paths exist: one through solid material, the other through the air space. These paths are parallel in the direction of heat flow, and the circuit representing such elements has two branches each consisting of a number of T-sections in series.

In such cases the assumption is made that the surfaces of the element are isothermal and that a common potential exists for the different branches at the surface nodes. In ideal one-directional heat flow conditions this may not be the case due to the difference in the resistance and capacitance of the different conduction paths; but in practice Buchberg* has shown that no appreciable error results from making such an assumption.

Each element in the conduction path is represented, in the circuit, by a number of lumps, or T-sections. To reduce the error arising from representing an essentially distributed property thermal system by a lumped property circuit (the lumping error) an adequate number of T-sections should be used for each element in the conduction path.

Nottage and Parmelee**, drawing on the analogy with A.C. transmission lines, suggested that the lumping error would be within 5% if the length of conduction path lumped is within an eighth of a wave length, where the wave length (λ) for periodic heat flow is given by

$$\lambda = 2\sqrt{\frac{\pi a}{f}} \text{ , ft} \quad \ldots\ldots\ldots\ldots \quad (4.1)$$

---

* loc.cit. see footnote 2.11.

** loc.cit. see footnote 2.10.

and where,

a = thermal diffusivity $(\frac{k}{\rho c})$ of the medium, ft^2/hr.

f = frequency, cycles/hr.

Substituting for $f = 1/24$, the length of the path to be lumped (l) would be

$$l \leq \frac{\lambda}{8} \leq 2.2 \quad a \quad , \text{ft} \quad \text{..........} \quad (4.2)$$

which, for example, for softwood, diffusivity = 0.0052, gives the length to be lumped

$l \leq 1.9$ inches

and for common brick, diffusivity = 0.0174, gives the length

$l \leq 3.5$ inches

It will, however, be seen from the circuit diagrams in Figs (1-6), that much finer lumping is possible, and has been used.

The circuit resistances for the simulator networks shown in Figs (1-6) are given in table 4.2. The circuit capacitances are given in table 4.3.

4.2.2 Insulation

4.2.2.1. Amount of insulation

The insulation studies are based on an insulating material with a conductivity of 0.25 Btu in./ft^2.hr.°F. This approximates the conductivity of expanded polystyrene or cork board* for example.

The thermal resistance (R_t) due to insulation is given by

$$R_t = \frac{l}{k} \quad {}^\circ F.hr.ft^2/Btu \quad \text{..........} \quad (4.3)$$

where l = thickness of insulation, inches.

k = conductivity of insulation, Btu in./ft^2.hr.°F.

---

* loc.cit. see Barned, footnote 4.3.

Table 4.2.

Circuit resistances, in Kilo-ohms.

Referring to Figs. 1-6, resistances taken from left to right.

Model T Fig. 1.	Top Row	195	408	423	245	268	333	65								
	Bottom Row	1364	2854	3250	4000	4000	4000	4000	2000	1875	2330	454				
Model H Fig. 2.	Top Row	76	76	78	157	364	363	157	78	76	76					
	Bottom Row	227	228	250	500	500	500	500	500	500	500	500	500	250	228	227
Model B Fig. 3.		25	50	50	50	50	—— 20 no. x 50 ——					50	50	25	57	57
Model L Fig. 4.	Top Row	95	96	268	554	285	178	179								
	Bottom Row	666	667	1875	4375	5000	5000	5000	5000	5000	5000	2500	1250	1250		
Model O Fig. 5.	Top Row	111	111	111	222	148	74	350	334	42	41					
	Bottom Row	333	334	334	667	429	190	190	190	190	190	190	97	125	125	
Model C Fig. 6.		83	84	50	100	79	55	55	55	55	55	29	57	57		

Table 4.3.

Circuit capacitances, in microfarads.

Referring to Figs. 1-6, capacitances taken from left to right.

Model T Fig. 1.	Top Row	2.07	0.93	0.01	1.4	0.7							
	Bottom Row	0.3	0.13	0.2	0.2	0.2	0.2	0.2	0.2	0.1			
Model H Fig. 2.	Top Row	1.69	1.7	1.7	0.01	1.7	1.7	1.69					
	Bottom Row	0.56	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.56
Model B Fig. 3.		2.0	2.0	2.0	2.0	2.0	—19 no. x 2.0—			2.0	2.0	2.0	2.25
Model L Fig. 4.	Top Row	0.76	1.4	0.02	0.93								
	Bottom Row	0.11	0.2	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.13		
Model O Fig. 5.	Top Row	0.66	1.5	1.5	3.5	3.5	0.01	1.5					
	Bottom Row	0.22	0.5	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	
Model C Fig. 6.		0.88	4.0	4.0	4.67	4.67	4.67	4.67	4.67	4.67	2.25		

If the quantity $(\frac{l}{k})$ is referred to as effective insulation resistance, then the thickness (l') of any insulating material of conductivity (k') which would give the same effective insulation resistance as a thickness (l) of the standard insulating material with conductivity $k = 0.25$ Btu in./ft.^{2}hr.^{o}F is

$$l' = \frac{lk'}{0.25} \text{ inches} \quad .... \quad (4.4)$$

Thus basing the studies on thicknesses of a standard insulating material (expanded polystyrene or cork board) does not impose any restriction on the scope of the studies. Using values of effective insulation resistance $(\frac{l}{k})$ gives a general 'scale' into which insulating materials of different conductivities can be fitted.

Since most insulating materials are of low density (expanded polystyrene, 1 lb./ft.^{3}; cork board* 5 lb./ft.^{3}) the insulation is assumed to have no heat capacity and is treated as pure resistance.**

The insulation is applied in increments of effective resistance $(\frac{l}{k}) = 2^o$F.hr.ft^2/Btu (equivalent to ½ inch increments of expanded polystyrene) from $\frac{l}{k} = 0$, to $\frac{l}{k} = 14$ ^{o}F.hr.ft^2/Btu (up to 3½ equivalent inches of expanded polystyrene), or until additional insulation ceases to affect the surface temperatures, of the building element, appreciably. A prediction of the diurnal variation of the surface temperatures of the element is obtained at each step.

For an element of area (A), from equation (4.3), the thermal resistance (R_t) due to insulation is

$$R_t = \frac{l}{kA} \quad {}^o\text{F.hr./Btu.}$$

---

* loc.cit. see Barned, footnote 4.3.

** Omitting the capacitance of insulation would produce the maximum error in the case of the timber wall, the lightest construction. The error for the maximum thickness of polystyrene (3½in.) used, would be in the order of 6%.

and the electrical resistance (R_e) is

$$R_e = R_t . N_R \quad \text{ohms}$$

where N_R, a scale factor, is given in Table 2.2 as 10^8.

The heat transfer areas of the different conduction paths, for the wall and roof elements, determined from the constructional details (Figs 1-6), are given in Table 4.4.

Table 4.4.

Heat transfer areas, for the different conduction paths, for wall and roof elements, ft^2.

Wall or roof	Air space path area	Solid path area
Timber wall Timber roof	175	25
Hollow block wall Hollow trough roof	150	50
Brick wall Solid concrete roof	-	200

The electrical resistances due to increments of effective insulation, are given for each of the above areas, in Table 4.5.

4.2.2.2. Position of insulation

The insulation is applied to the outside and inside of each wall and roof element. The alternative positions are shown in Figs (1-6). (In the case of the timber wall (model T) placing the insulation on either side of the studs was found to make little difference due to the small area of the stud

Table 4.5

Insulation resistances, for different amounts of insulation and heat transfer areas.

Effective resistance of insulation $(\frac{1}{k})$, $^{o}F.hr.ft.^{2}/Btu$	Equivalent inches of polystyrene ($k=0.25\ Btu/ft.^{2}hr.^{o}F$)	Insulation resistance, for relevant areas, in Kilo-ohms.				
		25 sq. ft.	50 sq. ft.	150 sq. ft.	175 sq. ft.	200 sq. ft.
2	$\frac{1}{2}$	8,000	4,000	1,133	1,143	1,000
4	1	16,000	8,000	1,667	2,286	2,000
6	$1\frac{1}{2}$	24,000	12,000	4,000	3,428	3,000
8	2	32,000	16,000	5,333	4,571	4,000
10	$2\frac{1}{2}$	40,000	20,000	6,667	5,714	5,000
12	3	48,000	24,000	8,000	6,857	6,000
14	$3\frac{1}{2}$	56,000	28,000	9,333	8,000	7,000

path compared to the air space path; so one position is shown).

4.2.3. Boundary conditions

4.2.3.1. Outdoor boundary conditions

Each wall construction was tested under four conditions of orientation and assumed to face east, west, north and south in turn. These are the orientations chosen in the last chapter (see section 3.1.5.1.) and for which the solar input was computed.

Apart from the fact that this choice corresponds to the traditional practice - in the Khartoum area - of orientating buildings to place them in the north-south summer wind stream, these four orientations also represent different conditions of solar input and present an interesting combination of outdoor conditions.

The west orientation presents a case where the solar input and the air temperature are maximum during the afternoon. The east orientation presents a condition where the solar input is maximum before noon, when the air temperature is still low. Only diffuse radiation is received on the west before noon, and on the east after noon. In the case of the north orientation a moderate solar input is symmetrical around noon. The south wall is in shade all day.

Roofs were assumed to be flat - which is in the tradition of building in dry climates - and were subjected to the solar input for a horizontal surface.

The solar input for all walls was computed using a solar absorptivity of 0.7 for the exterior surfaces. This value seems appropriate for brick, plaster and grey painted timber[6]. The solar absorptivity of all roofs was

---

6. Holden, T.S., and Greenland, J.J., "The coefficients of solar absorptivity and low temperature emissivity of various materials". Report No.6, Division of Building Research, C.S.I.R.O., Melbourne, 1951.

taken at 0.9 for built-up roofing*.

To generate the appropriate current in the analogue to represent these solar inputs, Function Generator resistances (see section 2.2.2.4.) are determined using equation (2.32) which gives a Function Generator resistance (R_g) for each input

$$R_g = \frac{V}{\alpha I_{max.} A N_q} \text{ , ohms}$$

where V = Function Generator potential = 200 volts.

A = area of surface = 200 sq. ft.

α = solar absorptivity (0.7, walls. 0.9, roofs)

N_q = scale factor (see Table 2.2) = $\frac{1}{2} \times 10^{-9}$.

$I_{max.}$ = highest hourly solar radiation falling on the surface (Table 4.6), Btu/hr.ft.^{2}

The maximum hourly solar input (I_{max}), for each orientation (as fitted in the Function Generators), and the Generator resistances used are given in Table 4.6.

Table 4.6

Function Generator resistances, for maximum hourly solar input, for different orientations, in Kilo-ohms.

Orientation	Horizontal	east	west	north	south
Maximum hourly input, Btu/hr.ft.^{2}	320	254	254	78	34
Function Generator resistance, Kilo-ohm	6,944	11,249	11,249	36,630	84,034

* Ibid.

The same outdoor temperature curve (Table 3.2) was taken to apply for all wall and roof elements.

Roofs were assumed to be in long wave radiative exchange with the sky at the temperatures given in Table 3.7, and walls with the environment at the temperatures given in Table 3.8.

4.2.3.2. Indoor boundary conditions

The tests were made for two indoor conditions. The first condition assumed was that the indoor air temperature is kept constant throughout the 24-hour cycle. The second was that the indoor air temperature varied as a function of the outdoor air temperature due to a ventilation coupling.

In the first case the temperature of the indoor air was assumed constant at 75°F. This is the temperature recommended by ASHRAE Guide* for air-conditioned buildings in hot arid climates.

A constant potential (given by equation 2.28) is connected to the node, in the wall or roof simulator network representing the inside surface through a resistance (R_i) representing the combined convective, radiative resistance to heat flow at the inside surface. The set-up is shown in Fig. 7.

In the second case the temperature of the indoor air is assumed to vary as a function of the outdoor air temperature. In this respect, it is imagined that the wall or roof element is part of an enclosure which suffers a certain number of air changes per hour.

The heat flux (w) due to the infiltration of outdoor air can be expressed,

---

* loc.cit., see footnote 4.1, chapter 27 on Air-conditioning.

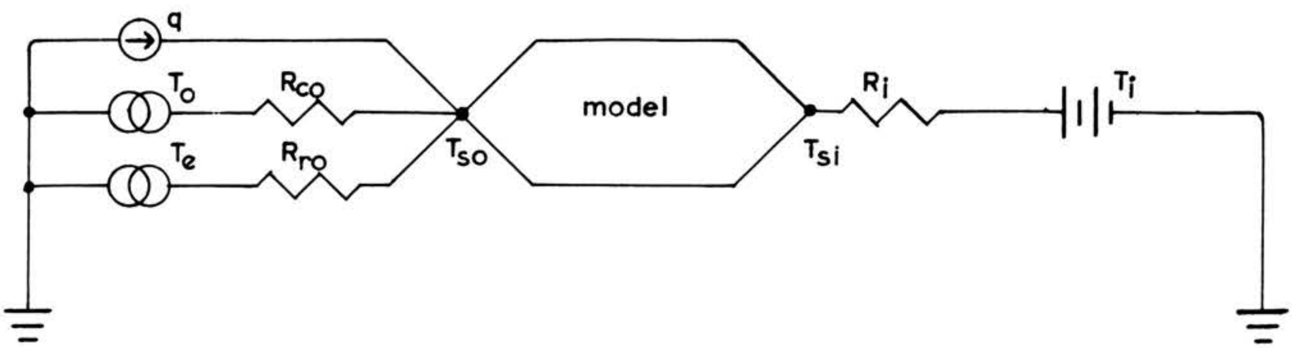


FIG. 7. BOUNDARY CONDITIONS — CONSTANT INSIDE AIR TEMPERATURE

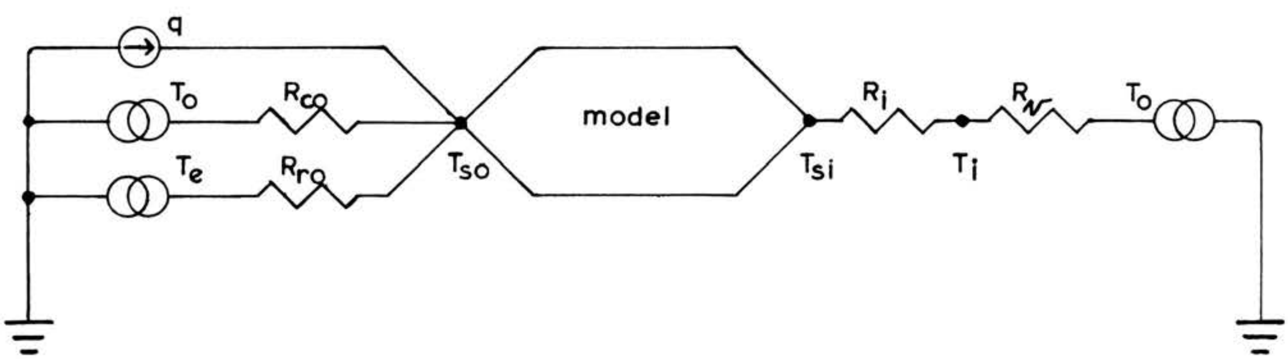


FIG. 8. BOUNDARY CONDITIONS — COUPLING TO OUTSIDE AIR TEMPERATURE

FIGS. 7. AND 8. BOUNDARY CONDITIONS FOR SINGLE WALLS OR ROOFS assuming perfect mixing of air, and temperature uniformity, as[7]

$$w = nC(T_o - T_i) \text{ , Btu/hr. } \quad \ldots\ldots\ldots\ldots \quad (4.5)$$

where, n = number of air changes per hour.

C = capacitance of room air, Btu/°F.

T_o, T_i = outdoor and indoor air temperatures respectively, °F.

But, $$C = \rho c V \text{ , Btu/°F. } \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.6)$$

where, ρ = density of air, lb./ft.^{3}

c = specific heat of air, Btu/lb.°F.

V = volume of room air, ft.^{3}.

The quantity $(\frac{1}{nC})$ is by definition (potential difference devided by heat flux) a resistance to heat flow due to ventilation. Assuming a room volume of 2000 cu.ft. and substituting for the density and specific heat of air (see Table 4.1) we get the thermal resistance

$$\frac{1}{nC} = \frac{1}{n \times 0.08 \times 0.24 \times 2000} = \frac{1}{38.4n} \sim \frac{1}{4on} \text{ , °F.hr./Btu.}$$

Multiplying by the scale factor (N_R), we get the electrical resistance (R_v) as

$$R_v = \frac{N_R}{4on} \text{ ohms } \quad \ldots\ldots\ldots\ldots (4.7)$$

where $N_R = 10^8$.

A number of studies were made to investigate the effect of changing the ventilation rate, on the indoor air temperatures. The results of these studies are reported in Appendix 3. From these studies it was found that at high values of (n), for example 10 air changes per hour, the indoor air temperature followed closely the outdoor air temperature variations. Using such a high ventilation rate would make the effects of insulation very

---

7. Parmelee, G.V., "Application of meteorological data to indoor climate in buildings". Bulletin of the American Meteorological Society, No. 6, Vol. 36, 1955.

difficult to detect. As the ventilation rate is reduced, however, a point is reached when further restriction in the ventilation rate does not affect the indoor air temperatures appreciably. This seemed to occur at a number of air changes per hour $n = 1$, for the elements tested. This value was adopted and the resistance (R_v) was determined using a value of $n = 1$.

From equation (4.7), therefore

$$R_v = \frac{10^8}{40} = 2500 \text{ Kilo-ohms.}$$

The outdoor temperature potential is connected to the indoor temperature node through the resistance (R_v). The indoor temperature node is, in turn, connected to the node representing the inside surface of the wall or roof element, through a resistance (R_i) representing the combined convective, radiative resistance to heat flow between the surface and the indoor air. The set-up is shown in Fig. 8.

4.3. Enclosure

4.3.1. Design of enclosure

A rectangular enclosure was considered, with the dimensions 20x10x11$\frac{3}{4}$ ft. The enclosure was assumed oriented with the ~~length~~ width (10ft.) on the north-south axis, and the ~~width~~ length (10ft.) on the ~~east-west~~ axis. Fairly heavy-weight construction is adopted as detailed below. Orthographic drawings of the enclosure are given in Fig. 14.

The north and south walls were assumed to have a window each, of 35 sq.ft. area. This gives solid, north and south, wall areas of 200 sq.ft., roof and floor areas of 200 sq.ft., and west and east wall areas of 117.5 sq.ft..

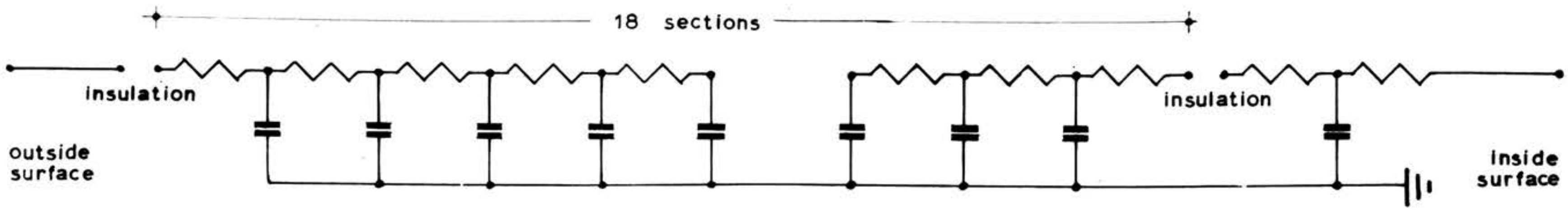
The east wall was assumed to have a door, 30 sq.ft. in area, which leaves a solid wall area of 87.5 sq.ft.

4.3.1.1. Constructional details

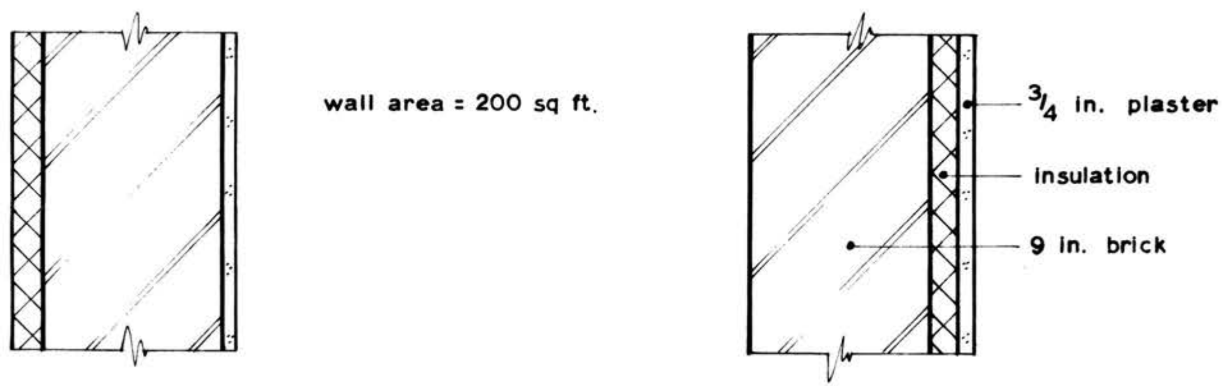
Roof :	A solid reinforced concrete roof, identical to model (C), described in section 4.2.1., was assumed.
North wall :	A hollow block wall, identical to model (H), described in section 4.2.1. was assumed.
South wall (Model S) :	A 9 in. common brick wall with $\frac{3}{4}$ in. plaster on the inside surface. (The simulator network for model B, described in Section 4.2.1. was adapted, by reducing T-sections representing $4\frac{1}{2}$ inches of brick, and used). See Fig. 9.
West wall (Model W) :	A $13\frac{1}{2}$ in. common brick wall with $\frac{3}{4}$ in. plaster on the inside surface. See Fig. 10.
East wall (Model P) :	A 9 in. common brick partition with $\frac{3}{4}$ in. plaster on both sides. Door, made of 2 in. solid softwood panel. See Fig. 11.
Floor (Model F) :	Compacted soil topped with 2 in. screed and 1 in. cement sand tiles. See Fig. 12.
Window Glass (Model G) :	$\frac{1}{4}$" plate glass on wooden frame. See Fig. 13.

4.3.1.2. Physical properties

Physical properties, not previously quoted in Table 4.1., are given for window and floor constructions in Table 4.7., from the same sources of reference. The simulator network's resistances for models S, W, P, F, and G are given in Table 4.8. The capacitances are given in Table 4.9.



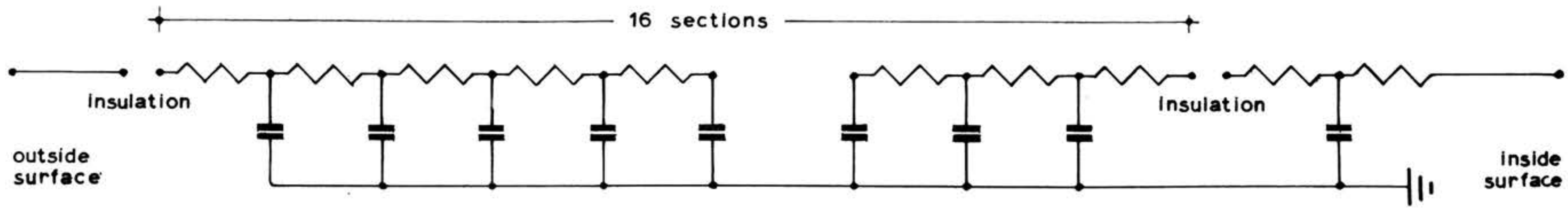
Electrical Model.



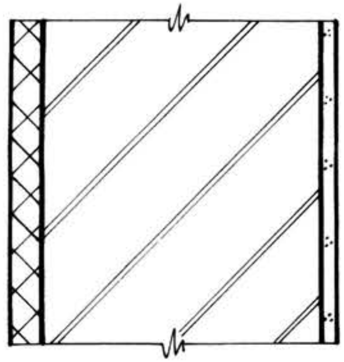
Section — insulation outside

Section — insulation inside

FIG. 9. BRICK WALL — MODEL "S".

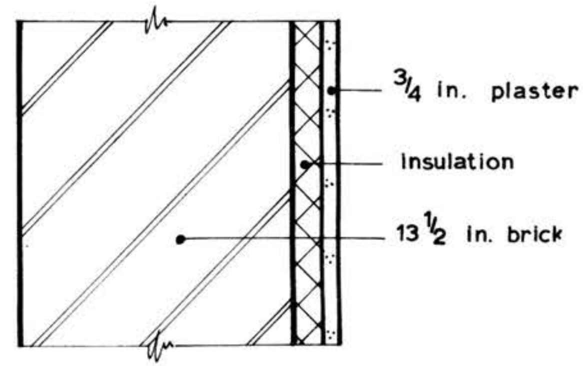


Electrical Model.



Section — outside insulation

wall area = 117.5 sq ft.



Section — inside insulation

FIG. 10. BRICK WALL — MODEL "W".

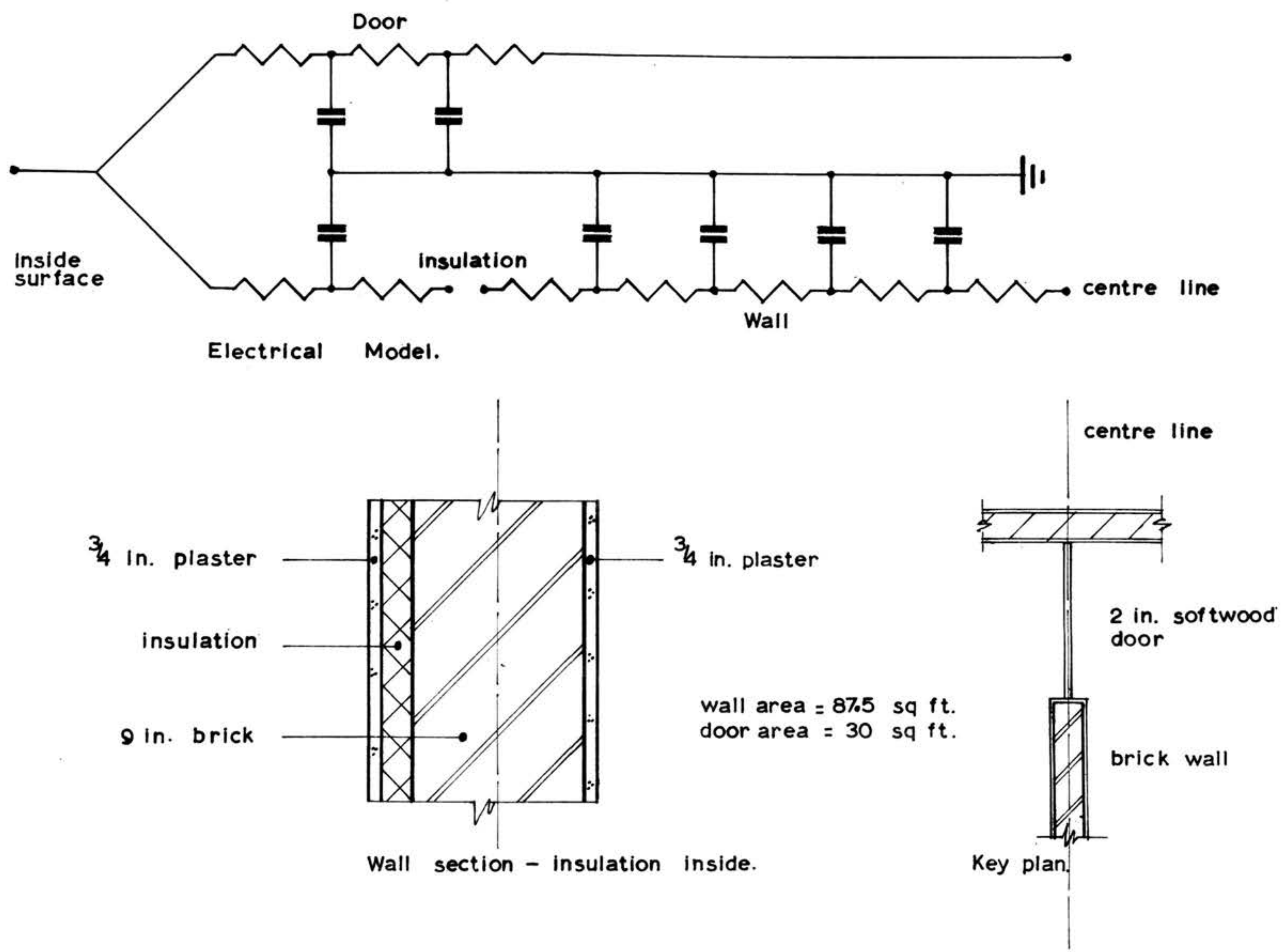
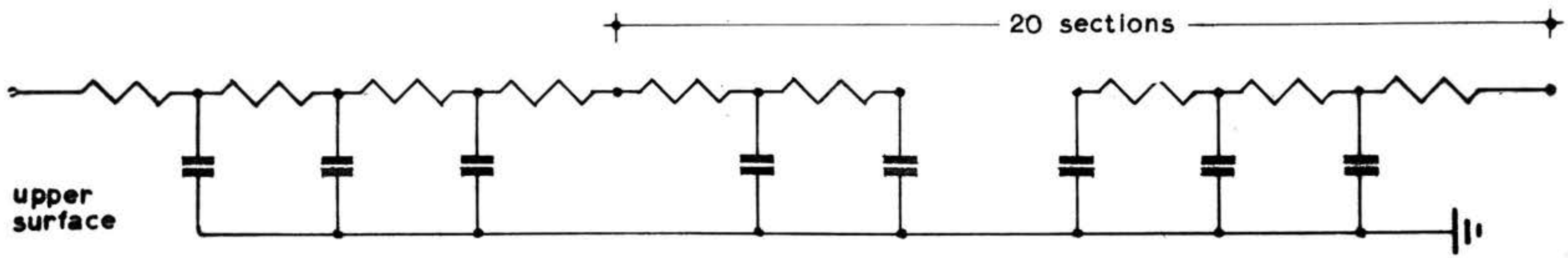
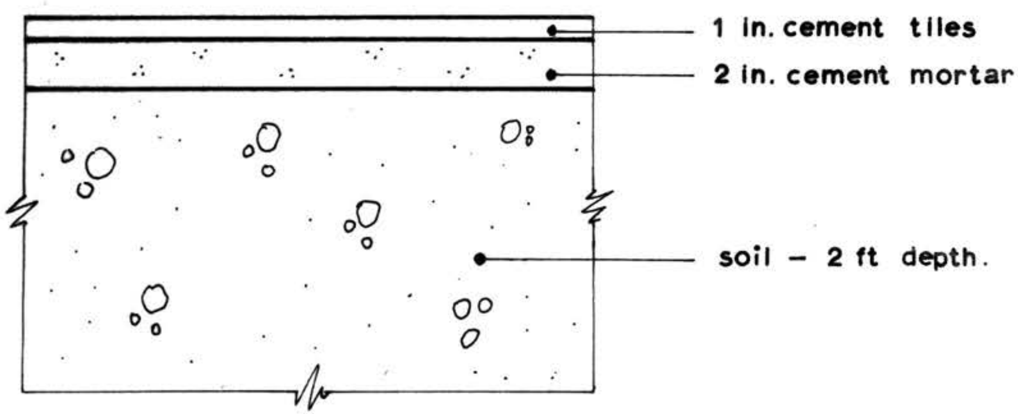


FIG. 11. BRICK PARTITION WITH DOOR - MODEL "P".



Electrical Model.

floor area = 200 sq ft.



Section

FIG. 12. FLOOR — MODEL "F".

window area = 35 sq ft.

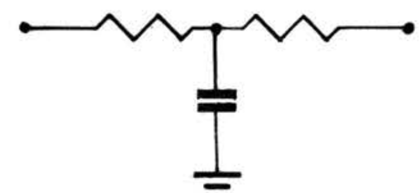


FIG. 13. WINDOW GLASS — MODEL "G".

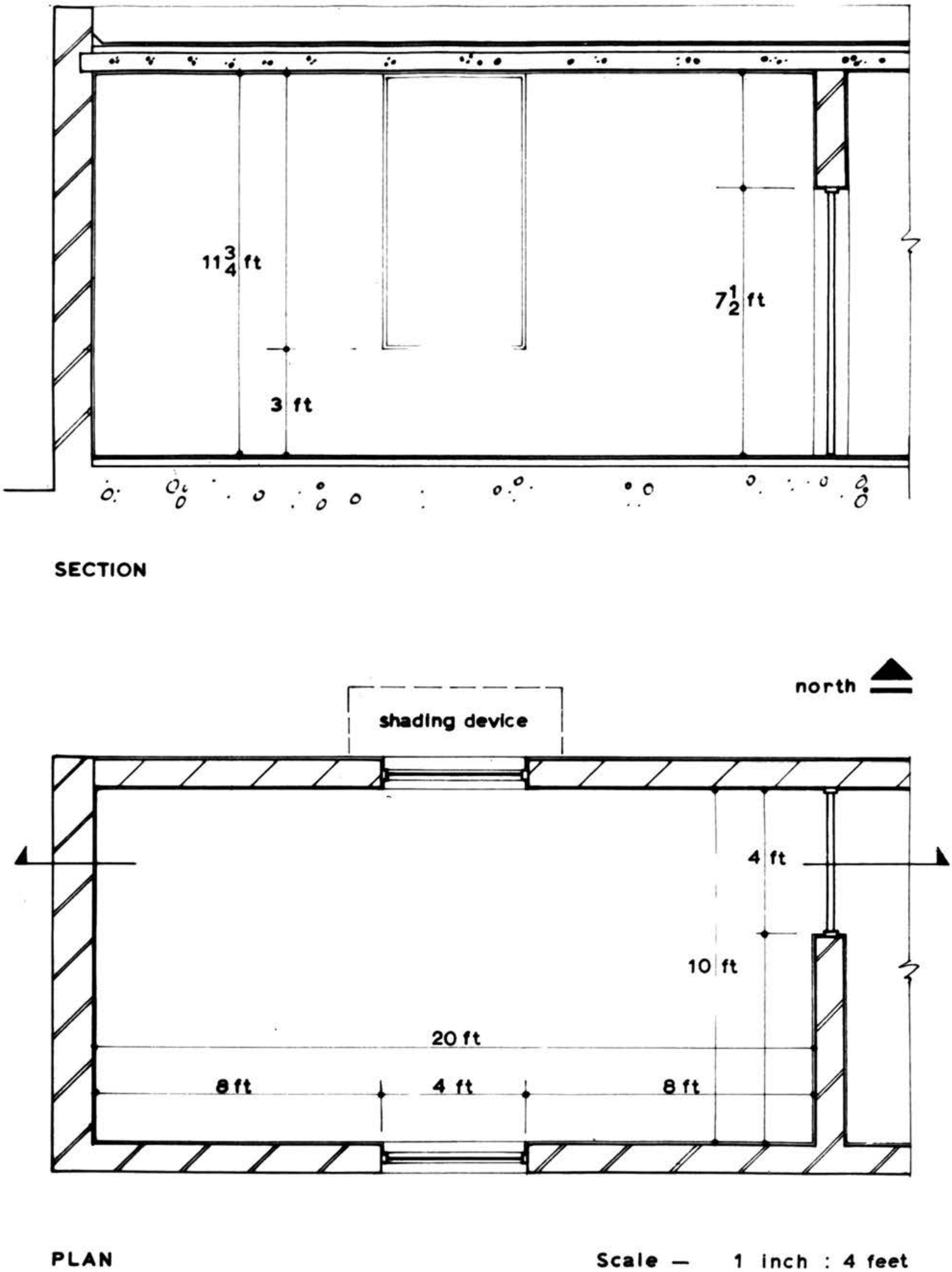


FIG. 14. PLAN AND SECTION OF TEST ENCLOSURE.

Table 4.7

Physical properties of additional materials used in construction of the enclosure.

Material	Density lb/cu.ft.	Specific heat Btu/lb.°F	Conductivity Btu in./ft.^{2}hr.°F.
$\frac{1}{4}$ in. plate glass	162	0.16	6.1
dry compacted soil	100	0.20	6.0
cement-sand screed cement-sand tiles	120	0.20	5.0

The behaviour of glass with respect to solar radiation differs from that of opaque building materials in that, as well as absorbing and reflecting some of the incident radiation, glass also transmits a portion of that radiation without absorbing it.

The transmission of glass to solar radiation depends on the chemical and physical characteristics of glass, and the wavelength and the angle of incidence of the radiation*.

The solar absorptivity of ordinary plate glass is quite low, and is given by Buchberg** at 0.02. Buchberg also gives the solar transmittance of plate glass as 0.88 for an angle of incidence $\theta = 0$, 0.53 at $\theta = 80^\circ$ and as nothing at grazing incidence. ASHRAE Guide gives the average spectral transmittance of $\frac{1}{4}$ in. plate glass as 0.80.

* loc.cit. See ASHRAE Guide, footnote 4.1.

** loc.cit. See Buchberg, footnote 2.11.

Table 4.8.

Circuit resistances, in Kilo-ohms.

Referring to Figs. 9-13, resistances taken from left to right.

Model S Fig. 9.		25	50	50	50	50	—— 11 no. x 50 ——	50	50	25	57	57
Model W Fig.10.		72	144	144	144	144	—— 9 no. x 144 ——	144	144	72	97	97
Model P	Top Row	1041	2082	1041								
Fig.11.	Bottom Row	130	130	128	257	257	257	129				
Model F Fig.12.		50	100	100	50	50	100	—— 16 no. x 100 ——	100	100	50	
Model G Fig.13.		58	59									

Table 4.9.

Circuit capacitances, in microfarads

Referring to Figs. 9-13, capacitances taken from left to right.

Model S Fig. 9.		2.0	2.0	2.0	2.0	2.0	—10 no. x 2.0—	2.0	2.0	2.0	2.25
Model W Fig.10.		2.0	2.0	2.0	2.0	2.0	— 8 no. x 2.0—	2.0	2.0	2.0	1.17
Model P Fig.11.	Top Row	0.16	0.16								
	Bottom Row	0.97	1.97	1.97	1.97	1.97					
Model F Fig.12.		4.0	4.0	4.0	4.0	4.0	—15 no. x 4.0———		4.0	4.0	4.0
Model G Fig.13.		0.19									

4.3.2. Boundary conditions

4.3.2.1. Outdoor boundary conditions

The north, south, west walls and the roof suffer solar inputs. The east wall was assumed to be a partition, not exposed to the Outdoor environment. The space on the other side of the partition was assumed to be at the same temperature as the enclosure, so that it could be assumed that no heat flow existed across the centre line of the partition.

The south wall, and therefore the south window are in shade all day and receive only diffuse radiation. On the north wall the direct component of solar radiation falls on the window at such large angles of incidence (see Table 3.4) that most of it is intercepted by the sides of the window opening; the window was, therefore, assumed to be shaded from the direct beam and taken to receive only diffuse radiation.

The generator resistances used to generate the appropriate current for each input are the same for the roof, north and south walls as those given in Table 4.6. The resistances for the west wall, north and south window glass* (assuming the window frame to be of negligible area) are given in Table 4.10, calculated using equation (2.32.).

Table 4.10

Generator resistances, for solar inputs, for west wall and north and south window glass, in Megohms.

West wall Area = 117.5 sq. ft.	North window glass Area = 35 sq. ft.	South window glass Area = 35 sq. ft.
19.147	15444	16807

* This represents only the absorbed portion of the radiation. In practice, these resistances are so large, the input due to absorbed radiation can be ignored.

The roof was assumed to be in long wave radiation exchange with the sky at the temperatures given in Table 3.7., and the walls and window glass with the environment at the temperatures given in Table 3.8.

The outdoor temperature curve given in Table 3.2 was assumed to apply for convection exchange with all exterior surfaces.

4.3.2.2. Indoor boundary conditions

Two indoor conditions were considered. The first condition was that the indoor air temperature was kept constant at 75°F for the diurnal cycle. The second condition was that indoor air temperature was coupled to the outdoor air temperature through a ventilation resistance. The ventilation resistance was calculated for 1 air change per hour, as discussed in section 4.2.3.2. and Appendix 3.

All inside surfaces exchange heat by convection with the indoor air, and by radiation with one another. The long wave radiation exchange between the surfaces is in proportion to the shape factors of the surfaces in relation to one another. The estimation of these shape factors and the radiative and convective surface coefficients is discussed in section 4.4.

The portion of diffuse solar radiation transmitted by window glass to the indoors should ideally be assumed to fall on the different room surfaces in proportion to their shape factors with respect to the windows[8]. The total input is, however, so small that such procedure would involve using very high radiative resistances resulting in great inaccuracies. The transmitted

---

8. Nottage, H.B., and Parmelee, G.V., "Circuit analysis applied to load estimating - Part II - Influence of transmitted solar radiation". Report No. 1529, ASHRAE Transactions, Vol. 61, 1955.

radiation was, therefore, assumed to fall on the floor of the enclosure.

The generator resistances for this input were computed using an equation, similar to equation 2.32, which gives the generator resistance (R_g) as

$$R_g = \frac{200}{t_g I_{max} AN_q} \text{ ohms} \quad \text{..........} \quad (4.8)$$

where, t_g = average spectral transmittance of plate glass = 0.8

and the other terms as defined for equation 2.32.

This gives generator resistances for the transmitted input from the north and south windows as follows

R_g (north window) = 386.1 Megohms.

R_g (south window) = 469.5 Megohms.

4.3.3. Insulation

An initial prediction of the temperatures of the indoor air and the surfaces of the elements forming the enclosure, is obtained before applying the insulation. Insulation is then applied, in turn, to the outside and inside of the wall and roof elements. In the case of the partition (east wall) the insulation is applied to the inside surface only. No insulation is applied to the floor.

In the first series of experiments on single wall and roof elements, the effect of varying the amount of insulation has been dealt with. In this series all wall and roof elements were assumed to have insulation with effective resistance $\frac{1}{k} = 4$, $^{\circ}$F.hr.ft.^{2}/Btu, the equivalent of 1 inch of expanded polystyrene. The interest here is in obtaining a comparison with the first series for different positions of the insulation.

Values of the insulation resistances for the roof, north and south walls (area = 200 sq.ft.) were as given in Table 4.5 for single elements. The insulation resistances for the west wall (area = 117.5 sq.ft.) and the east wall (area = 87.5 sq.ft.) were:

R (west wall) = 3404 Kilo-ohms

R (east wall) = 4571 Kilo-ohms.

4.3.4. Thermal circuit of the enclosure

The thermal circuit representing the enclosure is shown in Fig. 15. The electrical simulator network is shown in Fig. 16.

Each exterior surface is shown to be connected to the outdoor air temperature potential through a convection resistance R_{co}, and to the environmental temperature potential* through a radiation resistance R_{ro}. The values of these resistances for the different surfaces are given in section 4.4.

All surfaces are also shown to have a solar input (q) fed directly to the outside surface node. Although, the coupling is shown for window glass the magnitude of the solar input due to absorbed solar radiation is so small (see Table 4.10) that this input can be ignored and only the input due to transmitted radiation considered. This latter input is shown as an input to the floor.

All inside surfaces are shown to be connected to the indoor air temperature node (T_i) through convection resistances (R_{ci}), the values of which for different surfaces are given in section 4.4., and to other surfaces through

---

* The environmental temperature potential is different for the roof and walls. See section 4.3.2.1.

radiation resistances. The radiation network resistances are given in section 4.4.

The indoor air temperature node is shown connected, alternatively, to a time constant potential, when the indoor air temperature is assumed constant, and to the outdoor temperature potential through a ventilation resistance (R_v), when the indoor air temperature is assumed to fluctuate.

The indoor air temperature node is associated with a capacitance (C_a) representing the heat capacity of the indoor air, calculated from the expression

$$C_a = \rho c V N_C \quad \text{farads} \; .... \; (4.9)$$

which for : volume of air, $V = 2350$ ft.^{3}
density of air, $\rho = 0.08$ lb/ft.^{3}
specific heat of air, $c = 0.24$ Btu/lb.$^\circ$F.
and scale factor, $N_C = 10^{-8}$

gives $C_a = 0.45$ microfarads.

The simulator network for the partition (Fig. 11) represents only half the thickness of the construction. Since the temperature on both sides of the partition was assumed to be the same, no heat flow takes place across the centre line. The circuit, at that point, was left open.

At the inside surface of the partition the wall and the door panel surfaces were assumed to have the same potential. Since there were no external inputs, and since the door area is small, this assumption was thought justified as it simplifies the surface-to-surface radiation network.

Work, by Saad and Hendry[9], on ground temperatures at different depths in Khartoum, indicates that diurnal variations in temperature diminish rapidly

9. Saad, Shawki, and Hendry, A.W., "Measurements of soil temperatures and moisture contents". Civil engineering and public works review, Vol. 52, No. 609, March, 1957.

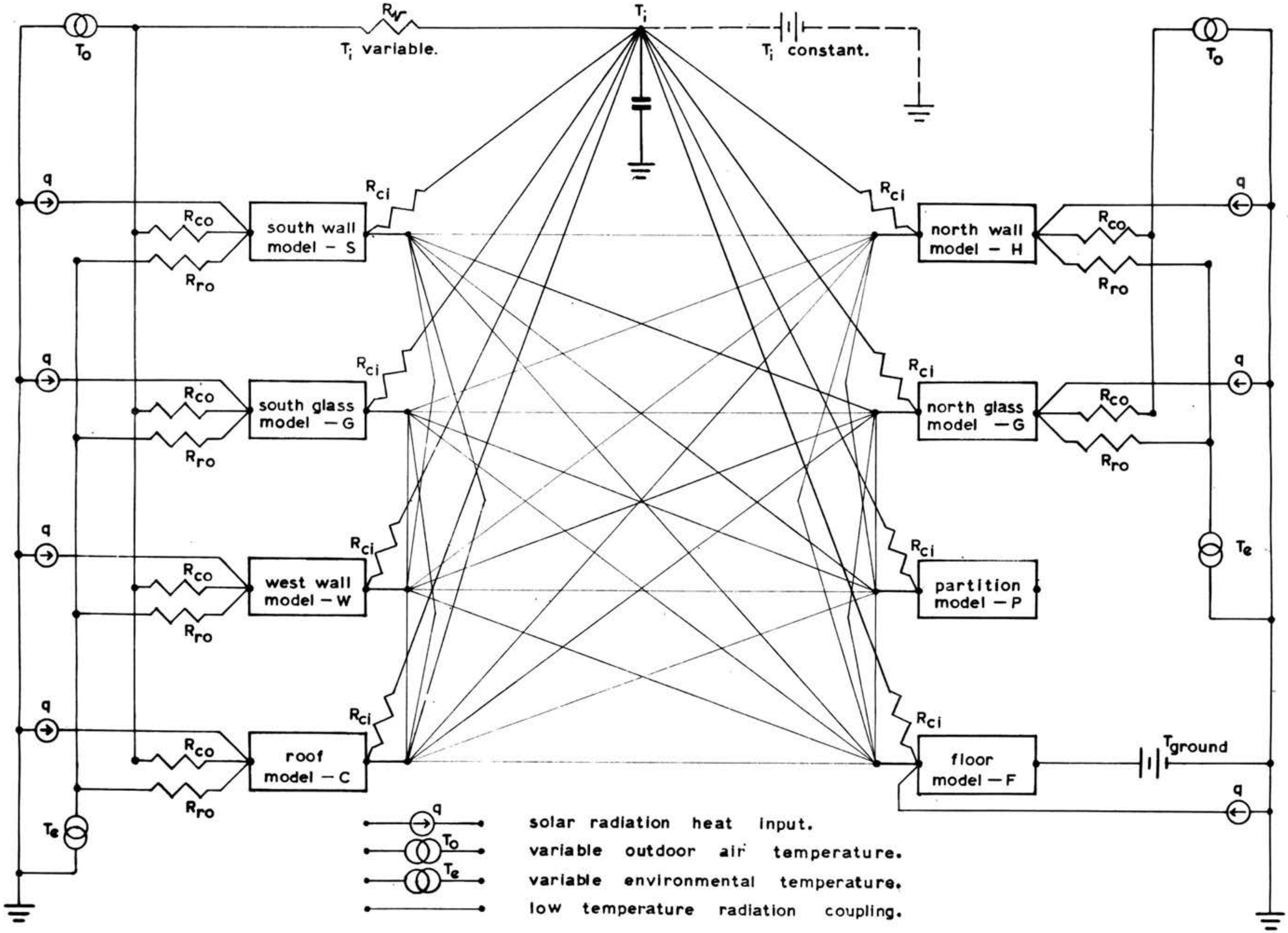


FIG. 15. THERMAL NETWORK FOR ENCLOSURE

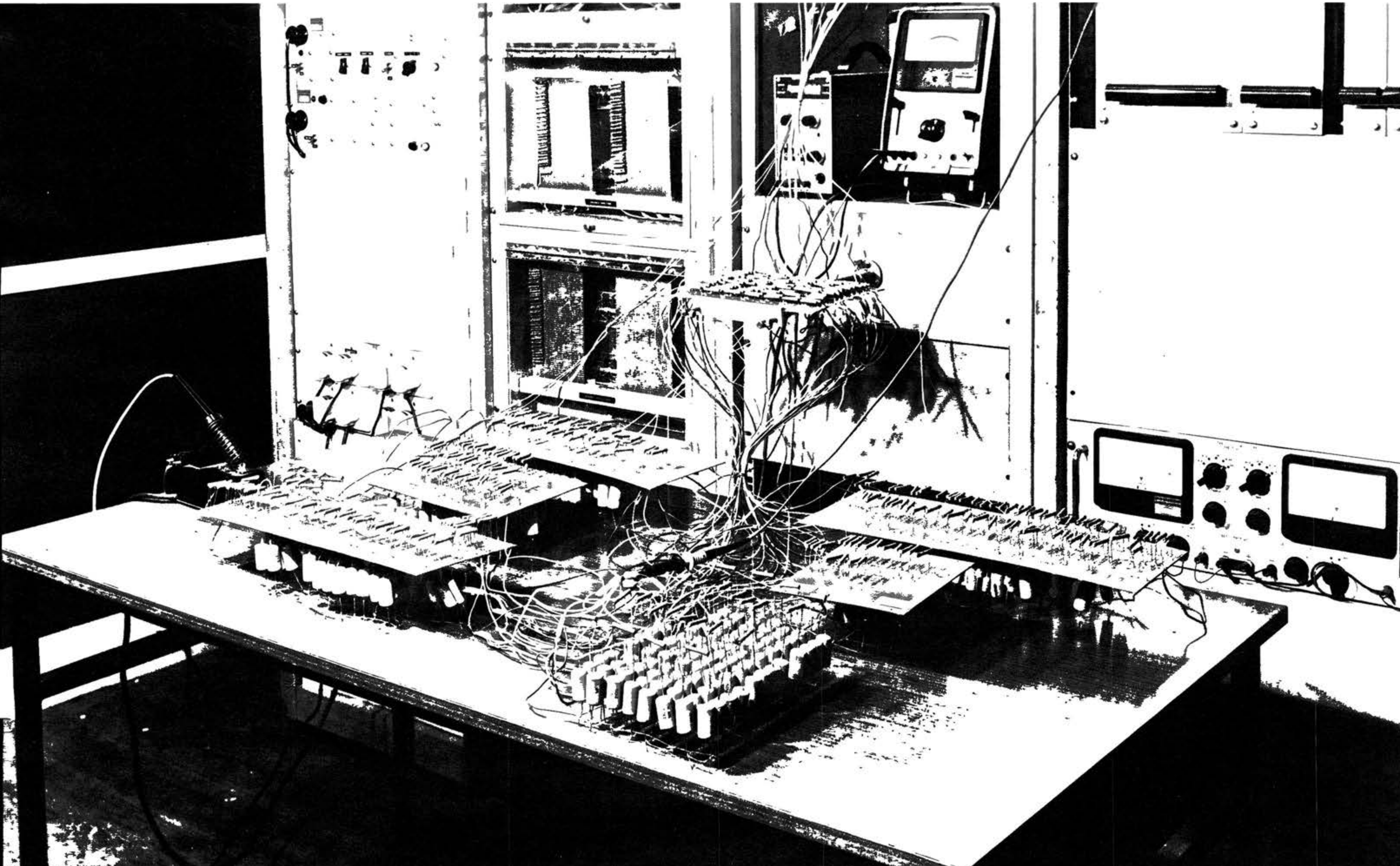


FIG. 16. ELECTRICAL MODEL OF ENCLOSURE.

with depth and become insignificant below 9 inches from the surface. In May the temperature two feet below the surface is indicated to be constant, for the diurnal cycle, at about 95°F.

The simulator network for the floor is made for two feet of soil depth (plus the finish) and in the thermal circuit, a time constant potential (T_{ground}) is shown to exist under that depth of ground, representing 95°F.

In drawing the thermal circuit of the enclosure the assumption is made that corner effects are negligible.

4.4. Surface Coefficients

The surface coefficients requiring determination are:

1 - Outside surface convection coefficient (h_{co}) for heat transfer between an outside surface and the outdoor air.

2 - Outside surface radiation coefficient (h_{ro}) for long wave radiation exchange between an outside surface and the environment.

3 - Inside surface convection coefficient (h_{ci}) for heat transfer between an inside surface and the indoor air.

4 - Inside surface radiation coefficient ($h_{r(1-2)}$) for long wave radiation exchange between different surfaces of an enclosure.

5 - Inside surface combined convection-radiation coefficient (h_i) for heat transfer by convection and radiation at the inside surface of an element.

Although each of these coefficients may vary in magnitude with a number of factors, as was discussed in sections 1.1.4. and 1.1.5., they are assumed, for the purpose of this investigation, to be constant for the 24-hour cycle.

4.4.1. Outside surface convection coefficient

It has been argued in section 1.1.4.3. that this coefficient can be taken as the sum of the natural and forced convection coefficients. Its value, therefore, depends on the geometry and orientation of the surface, the temperature difference between the surface and air and on the wind movement over the surface. It has, however, been indicated that the contribution due to natural convection may be very small compared to the forced convection component, except at very low wind speeds.

Roux* produced curves showing the variation of the coefficient with wind speed, and has shown that, although the day and night time values of the convection coefficient vary slightly for the same wind speed, a curve representing the average variation with wind speed can be established, from which the coefficient can be estimated with a reasonable degree of accuracy.

This curve, for the design wind speed of 5.5 M.P.H. adopted in section 3.1.7, gives a convection coefficient (h_{co}) at the outside surface as 2.0 Btu/ft^2.hr.$^\circ$F.

This value was used, in this text, and assumed to apply for all vertical and horizontal surfaces.

A convective resistance (R_{co}) appears in the thermal circuit, the magnitude of which is given by equation (2.30), section 2.1.5.3., as

$$R_{co} = \frac{1}{h_{co} A} \cdot N_R \text{ ohms.}$$

where, A = area of surface, ft^2.

N_R = scale factor = 10^8.

---

* loc.cit. see footnote 1.16.

4.4.2. Outside surface radiation coefficient

This coefficient was given, by equation (1.26), as

$$h_{ro} = \epsilon\sigma(\tau_{so}^2 + \tau_e^2)(\tau_{so} + \tau_e), \text{ Btu/ft}^2\text{.hr.}^\circ\text{F.}$$

where, σ = Stephan-Boltzmann constant = 0.173×10^{-8}, Btu/ft^2.hr.$^\circ$R^4.

ϵ = emissivity of surface.

τ_{so}, τ_e = absolute temperatures of surface and environment, respectively, $^\circ$R.

Since the surface and environmental temperatures vary continuously an approximation is necessary if one value of the coefficient is to be found. For small temperature differences we can write

$$h_{ro} = 4\epsilon\sigma\tau_m^3, \text{ Btu/ft}^2\text{.hr.}^\circ\text{F.} \quad \ldots\ldots\ldots\ldots (4.10)$$

where,

$$\tau_m = \frac{\tau_{so} + \tau_e}{2}, \ ^\circ\text{R} \quad \ldots\ldots\ldots\ldots (4.11)$$

If it can further be assumed that

$$\tau_m = \tau_o \quad \ldots\ldots\ldots\ldots (4.12)$$

where, τ_o = absolute average temperature of the outdoor air, $^\circ$R.

then equation (4.10) becomes

$$h_{ro} = 4\epsilon\sigma\tau_o^3 \quad \text{Btu/ft}^2\text{.hr.}^\circ\text{F.} \quad \ldots\ldots\ldots\ldots (4.13)$$

Taking the average outdoor temperature (see Table 3.2) as 97°F = 556 $^\circ$R, and the emissivity of all exterior surfaces* to be 0.9, we have

$$h_{ro} = 1.07 \sim 1.1 \quad \text{Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

---

* This is a reasonable value for the building material considered, and glass. See Holden, footnote 4.6.

It has been argued by Roux* that the sum of the outside surface convection and radiation coefficients can be taken as constant at 3.5 Btu/ft^2.hr.$^\circ$F., for summer conditions in South Africa. The radiation coefficient determined by Roux was computed by dividing the total long wave heat exchange by the temperature difference between the surface and the outdoor air. As such it is different from the radiation coefficient used in this text which is referred to the temperature difference between the surface and the environment 'seen' by the surface.

The value of h_{ro} = 1.1 Btu/ft^2.hr.$^\circ$F. was used in this investigation and assumed to apply for all outside surfaces.

A radiative resistance (R_{ro}) appears in the thermal circuit, the magnitude of which is given by equation (2.30) as

$$R_{ro} = \frac{1}{h_{ro}A} \cdot N_R \quad \text{ohms.}$$

where, A = area of surface, ft^2.

N_R = scale factor = 10^8.

4.4.3. Inside surface convection coefficient

For low levels of air movement the inside surface convection coefficient (h_{ci}) may be estimated on the basis of natural convection. Its value would then be a function of the surface geometry and orientation, and the temperature difference between the surface and air.

It will be seen from the discussion in section 1.1.4.3. that the coefficient can be expressed, for large surfaces, (equation 1.34) as

---

* loc.cit., see footnote 1.16.

$$h_{ci} \text{ (vertical surfaces)} = 0.3(\Delta T)^{\frac{1}{4}}, \text{Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

$$h_{ci} \text{ (horizontal surfaces, facing up)} = 0.4(\Delta T)^{\frac{1}{4}}, \text{Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

$$h_{ci} \text{ (horizontal surfaces, facing down)} = 0.2(\Delta T)^{\frac{1}{4}}, \text{Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

where ΔT = temperature difference between surface and air, °F.

However, because the choice of (ΔT) is usually arbitrary when choosing coefficient values, various values are reported in the literature*.

In this text the coefficient is estimated using the above expressions, and assuming an average temperature difference between surfaces and the indoor air of 10°F. This gives

$$h_{ci} \text{ (vertical surfaces)} = 0.50 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

$$h_{ci} \text{ (floor)} = 0.70 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

$$h_{ci} \text{ (ceiling)} = 0.35 \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

Convective resistances (R_{ci}) appear in the thermal circuit of the enclosure, the magnitudes of which are given by equation (2.30) as

$$R_{ci} = \frac{1}{h_{ci}A} \cdot N_R \quad \text{ohms.}$$

where, A = area of surface, ft.²

N_R = scale factor = 10^8.

4.4.4. Inside surface radiation coefficient

The coefficient for long wave radiation exchange between two surfaces of an enclosure that 'see' one another, has been given by equation (1.47) as

$$h_{r(1-2)} = \sigma F_e F_{1-2}(\tau_1^2 + \tau_2^2)(\tau_1 + \tau_2) \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

---

* See, for example Nottage et al., footnote 2.10, and Buchberg, footnote 2.9.

where, σ = Stephan-Boltzmann constant = 0.173×10^{-8}, Btu/ft^2.hr.$^\circ$R^4.

F_e = emissivity factor for the two surfaces with respect to one another.

F_{1-2} = shape factor for surface (2) with respect to surface (1).

τ_1, τ_2 = absolute temperatures of surface (1) and (2), respectively, $^\circ$R.

For small temperature differences, the expression can be written as

$$h_{r(1-2)} = 4\sigma F_e F_{1-2} \tau_m^3 \text{, Btu/ft}^2\text{.hr.}^\circ\text{F. ....(4.14)}$$

where,

$$\tau_m = \frac{\tau_1 + \tau_2}{2} \text{, }^\circ\text{R .... (4.15)}$$

The average surface temperature τ_m is however, difficult to estimate since various indoor conditions are assumed, with the air temperature taken to be constant at 75°F sometimes, and sometimes assumed to vary due to a coupling to the outdoor air temperature. The value of the coefficient is not, however, very sensitive to small temperature changes. Assuming the average temperature of inside surfaces to be equal to the average outdoor air temperature (97°F), as would approximate the condition when a ventilation coupling is assumed gives

$$h_{r(1-2)} = 1.19 F_e F_{1-2} \text{, Btu/ft}^2\text{.hr.}^\circ\text{F .... (4.16)}$$

Assuming the average temperature of the inside surfaces to be (80°F), as would approximate the condition when the indoor air temperature is constant at 75°F, gives

$$h_{r(1-2)} = 1.09 F_e F_{1-2} \text{, Btu/ft}^2\text{.hr.}^\circ\text{F. .... (4.17)}$$

the difference being less than 10% of the value of the coefficient. Since, however, interest really lies in the worst conditions when the temperature of the surfaces is expected to be appreciably high the value of the coefficient given by equation (4.16) is adopted.

The emissivity factor has been given by equation (1.48) for parallel surfaces, and equation (1.49) for perpendicular surfaces. Assuming all surfaces to have an emissivity of 0.9, we have

$$F_e \text{ (parallel surfaces)} = 0.818 \quad \ldots\ldots\ldots\ldots (4.18)$$

$$F_e \text{ (perpendicular surfaces)} = 0.81 \quad \ldots\ldots\ldots (4.19)$$

which gives

$$h_{r(1-2)} \text{ (parallel surfaces)} = 0.97\, F_{1-2},\ \text{Btu/ft.}^2\text{hr.}^\circ\text{F} \ \ldots\ (4.20)$$

$$h_{r(1-2)} \text{ (perpendicular surfaces)} = 0.96\, F_{1-2},\ \text{Btu/ft.}^2\text{hr.}^\circ\text{F} \ \ldots\ (4.21)$$

The shape factor (F_{1-2}) has been given by equation (1.50), section 1.1.5.2. The use of equation (1.50) for the evaluation of the shape factor involves a considerable amount of analytical work and can be done satisfactorily only for simple cases. Hottel*, however, produced curves from which shape factors can be estimated for parallel and perpendicular planes of different dimensional ratios. These curves were used to estimate shape factors, for different elements in the enclosure.

The shape factors, the radiation coefficients evaluated from equation (4.20) and (4.21), and the resistances for the surface-to-surface radiation network of the enclosure, are given in Table 4.11.

The resistances are determined, using equation (2.30), as

$$R_{r(1-2)} = \frac{1}{A_1 h_{r(1-2)}} \cdot N_R \quad \text{ohms}$$

where, A_1 = area of surface (1), ft.^{2}

N_R = scale factor = 10^8.

---

* loc.cit. see Hottel, footnote 1.26, also Billington, footnote 1.23.

For the purpose of Table 4.11, the different surfaces of the enclosure are designated by letters as follows:

west wall	:	w
north wall	:	n
east wall	:	e
south wall	:	s
ceiling	:	c
floor	:	f
north glass	:	a
south glass	:	b

Table 4.11

Shape factors F_{1-2}, radiation coefficients $h_{r(1-2)}$, and radiation network resistances $R_{r(1-2)}$, for the enclosure

Direction†	Surface Symbol	Area, ft.²	Shape factor	Coefficient Btu/ft.² hr. °F.	Resistance Kilo-ohms
F_{w-n}	w	117.5	0.20	0.1896	4,480
F_{w-e}			0.09	0.0873	9,750
F_{w-s}			0.20	0.1896	4,480
F_{w-f}			0.22	0.2086	4,080
F_{w-c}			0.22	0.2086	4,080
F_{w-a}			0.04	0.0379	22,460
F_{w-b}			0.04	0.0379	22,460

Direction	Surface Symbol	Surface Area, ft^2	Shape factor	Coefficient $Btu/ft^2 hr. {}^oF.$	Resistance Kilo-ohms
F_{n-e}	n	200	0.12	0.1138	4,390
F_{n-s}			0.29	0.2813	1,780
F_{n-f}			0.22	0.2086	2,400
F_{n-c}			0.22	0.2086	2,400
F_{n-b}			0.04	0.0379	13,190
F_{e-s}	e	117.5	0.20	0.1896	4,480
F_{e-f}			0.22	0.2086	4,080
F_{e-c}			0.22	0.2086	4,080
F_{e-a}			0.04	0.0379	22,460
F_{e-b}			0.04	0.0379	22,460
F_{s-f}	s	200	0.22	0.2086	2,400
F_{s-c}			0.22	0.2086	2,400
F_{s-a}			0.04	0.0379	13,190
F_{f-c}	f	200	0.26	0.2522	1,980
F_{f-a}			0.035	0.0332	15,060
F_{f-b}			0.035	0.0332	15,060
F_{c-a}	c	200	0.035	0.0332	15,060
F_{c-b}			0.035	0.0332	15,060
F_{a-b}	a	35	0.09	0.0873	48,630

+ F_{w-n}, implies that portion of radiation emitted by surface (w) which falls on Surface (n) and so on.

4.4.5. Inside surface combined convection-radiation coefficient

When wall and roof elements are tested individually and no surface-to-surface radiation coupling exists, the surface coefficient of heat transfer is estimated to allow for radiation exchange.

Roux* suggested that an 'artificial' radiation coefficient (h_{ri}) be added to the inside surface convection coefficient (h_{ci}) to give an overall surface coefficient (h_i). This combined coefficient must then be defined as the rate of heat transfer by convection and long wave radiation, at the inside surface, for unit area of the surface, in unit time, for unit temperature difference between the surface and the indoor air.

From equation (1.45), (taking the shape factor of all surfaces with respect to the surface under consideration to be equal to unity, by equation (1.52)) this 'artificial' radiation coefficient can be written as

$$h_{ri} = \frac{\sigma F_e (\tau_1^4 - \tau_2^4)}{T_1 - T_i}, \text{ Btu/ft}^2\text{.hr.}^\circ\text{F} \quad \ldots\ldots\ldots\ldots (4.22)$$

where,

σ = Stephan-Boltzman constant = 0.173×10^{-8}, Btu/ft^2.hr.$^\circ$R^4

F_e = emissivity factor.

τ_1, τ_2 = absolute temperature of surface (1) and absolute average temperature of other surfaces 'seen' by surface (1), $^\circ$R.

T_1, T_i = surface and indoor air temperatures respectively, $^\circ$F.

Equation (4.22) can be approximated to be

$$h_{ri} = \frac{4\sigma F_e \tau_m^3 (T_1 - T_2)}{T_1 - T_i}, \text{ Btu/ft}^2\text{.hr.}^\circ\text{F} \quad \ldots\ldots (4.23)$$

---

* loc.cit. see footnote 1.13.

where,

$$\tau_m = \frac{\tau_1 + \tau_2}{2}\ , \ ^\circ R \ \ldots\ldots\ldots\ldots (4.24)$$

Equation (4.23) brings out the dependence of the 'artificial' radiation coefficient, in magnitude and sense, on the quantity

$$\frac{T_1 - T_2}{T_1 - T_i}$$

It has been shown by Roux* that although the value of the 'artificial' radiation coefficient can vary between wide limits and can be positive or negative, this coefficient forms a fairly large fraction of the overall surface coefficient and should not be ignored.

Equations (4.20) and (4.21) give an indication as to the possible magnitude of such a coefficient. These equations refer to a coefficient expressed in terms of the temperature difference between two inside surfaces, but since the difference between these temperatures and the indoor air temperature is not usually large enough to drastically affect the value of the coefficient, an 'artificial' radiation coefficient can be assumed to be of comparable order.

If we accept a value of h_{ri} = 0.95 Btu/ft.^{2}hr.$^\circ$F. (since in equation (4.20) and (4.21), F_{1-2} can be taken as unity for the sum of surfaces) we have the combined convection-radiation coefficient, at the inside surface of a wall or roof as:

$$h_i \text{ (walls)} = 0.50 + 0.95 = 1.45, \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

$$h_i \text{ (ceilings)} = 0.35 + 0.95 = 1.30, \text{ Btu/ft.}^2\text{hr.}^\circ\text{F.}$$

Comparable coefficients quoted by van Straaten**, as recommended in America, are 1.46 and 1.08 respectively, and in South Africa, are 1.65 and 1.20 respectively.

---

* loc.cit., see footnote 1.13.

** loc.cit., see footnote 1.23.

A surface resistance (R_i) appears in the thermal circuit of single elements (Figs. 7 and 8) for heat transfer by convection and radiation at the inside surface, the magnitude of which is given by equation (2.30) as

$$R_i = \frac{1}{h_i A} \cdot N_R \text{ ohms}$$

where, A = area of surface, $ft.^2$

N_R = scale factor = 10^8.

CHAPTER 5

RESULTS ANALYSIS

Two types of results are presented. Firstly, those dealing with the initial prediction of temperatures of uninsulated walls and roofs and the uninsulated enclosure. Secondly those dealing with the effect of varying the amount and position of insulation, on these temperatures. Results of ventilation studies are reported in Appendix 3.

5.1. PREDICTION OF TEMPERATURES OF UNINSULATED BUILDING ELEMENTS.

5.1.1. Results.

Figs. (1-8) refer to the timber wall, Figs. (9-16) to the hollow block wall and Figs. (17-24) to the brick wall, in the first test series on single elements.

For each wall, two figures are given for each of four orientations of the wall: one for a constant indoor air temperature, the other for the case where the indoor air temperature is coupled to the outdoor air temperature through a (1 air change per hour) ventilation coupling.

Similarly two figures, Figs. (25-26), are given for the timber roof, two, Figs. (27-28), for the hollow trough roof, and two, Figs. (29-30), for the solid concrete roof, in the first test series on single elements.

The results for the enclosure are given in Figs. (31-32) for the roof, Figs. (33-34) for the west wall, Figs. (35-36) for the north wall, Figs. (37-38) for the south wall, Figs. (39-40) for north and south window glass, and Figs. (41-42) for the floor and partition.

Each of the Figs. (1-40) shows a curve of the outside surface temperature (T_{so}) of the element, a curve of the inside surface temperature (T_{si}), a curve of the outdoor air temperature (T_o), and a curve for the indoor air temperature

(T_i). The north window glass and the south window glass have identical curves and are given together in Figs. (39-40). In the case of the floor and partition, only inside surface, and air, temperatures are given in Figs. (41-42).

A summary of the maximum and minimum surface temperatures, for each of the figures presented, together with the times of occurence of these temperatures, is presented in Tables 5.1. - 5.4.

Table 5.1. refers to the odd numbered Figs. (1-29) which represent cases with variable indoor air temperature. Table 5.2. refers to the even numbered Figs. (2-30) which represent cases with constant indoor air temperature. Table 5.3. refers to odd numbered Figs. (31-41) for the enclosure, when the indoor air temperature is variable, and Table 5.4. to even numbered Figs. (32-42) for the enclosure, when the indoor air temperature is assumed constant.

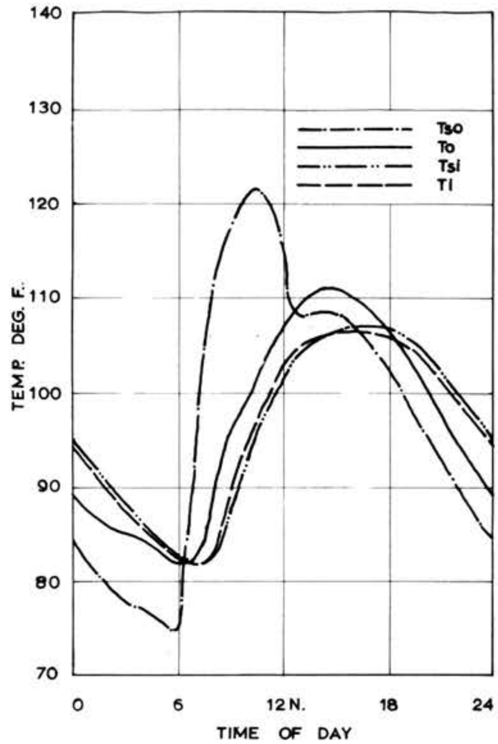


FIG. 1. EAST FACING TIMBER WALL
VARIABLE INSIDE AIR TEMPERATURE

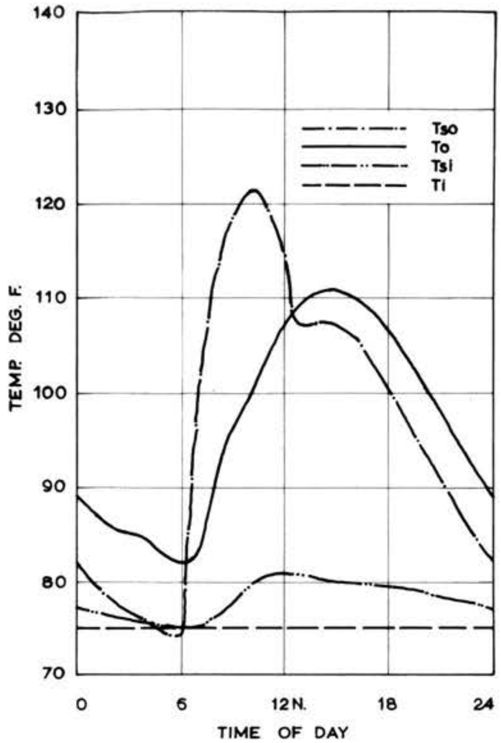


FIG. 2. EAST FACING TIMBER WALL
CONSTANT INSIDE AIR TEMPERATURE

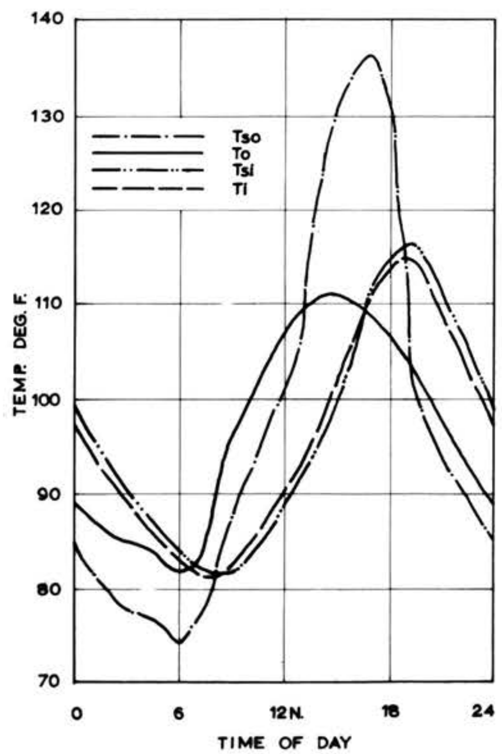


FIG. 3. WEST FACING TIMBER WALL
VARIABLE INSIDE AIR TEMPERATURE

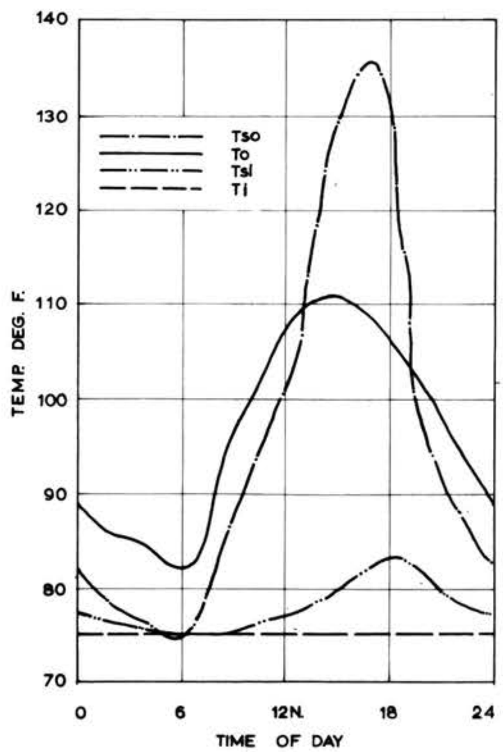


FIG. 4. WEST FACING TIMBER WALL
CONSTANT INSIDE AIR TEMPERATURE

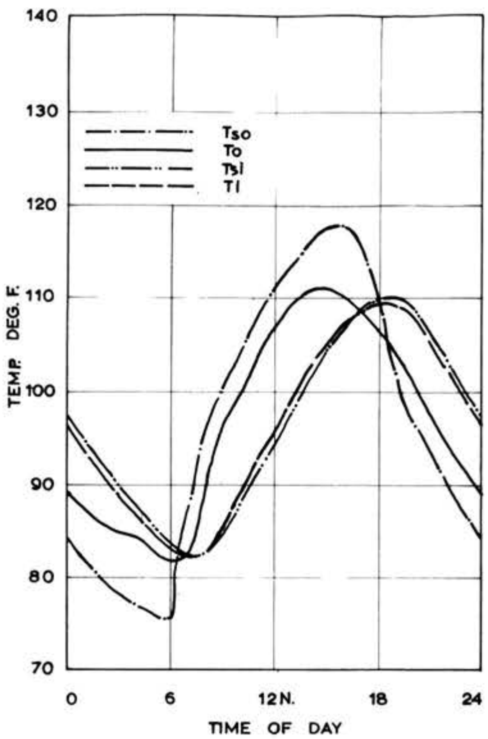


FIG. 5. NORTH FACING TIMBER WALL
VARIABLE INSIDE AIR TEMPERATURE

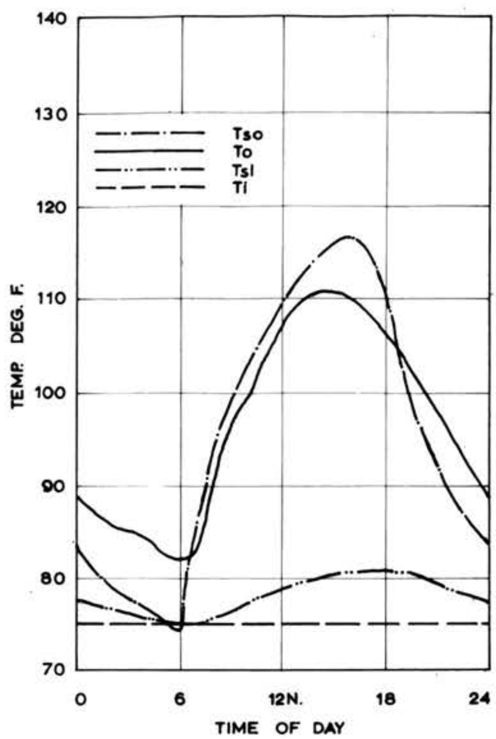


FIG. 6. NORTH FACING TIMBER WALL
CONSTANT INSIDE AIR TEMPERATURE

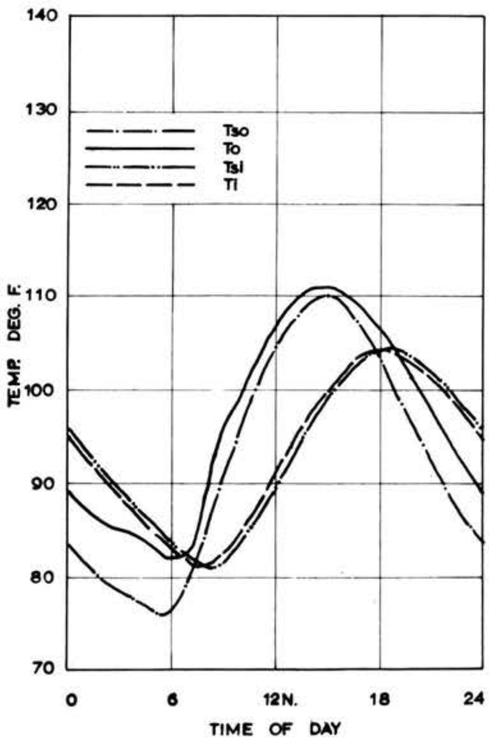


FIG. 7. SOUTH FACING TIMBER WALL
VARIABLE INSIDE AIR TEMPERATURE

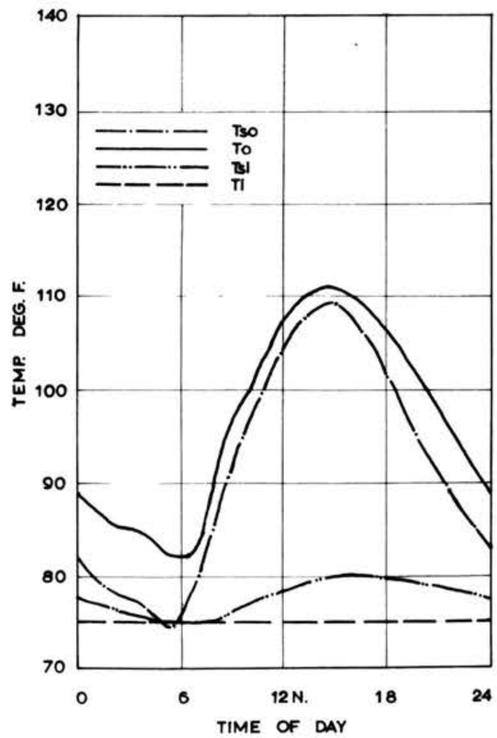


FIG. 8. SOUTH FACING TIMBER WALL
CONSTANT INSIDE AIR TEMPERATURE

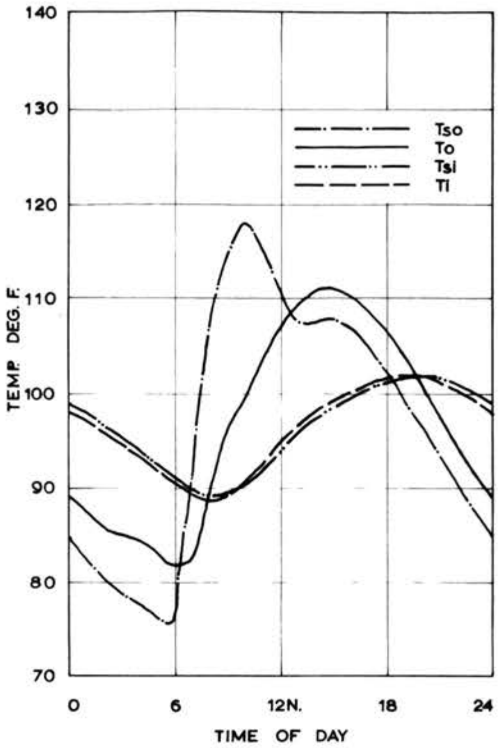


FIG. 9. EAST FACING H. BLOCK WALL VARIABLE INSIDE AIR TEMPERATURE

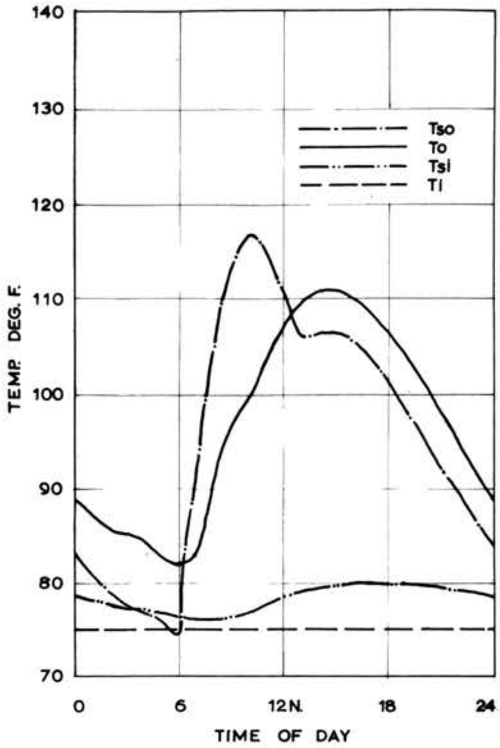


FIG. 10. EAST FACING H. BLOCK WALL CONSTANT INSIDE AIR TEMPERATURE

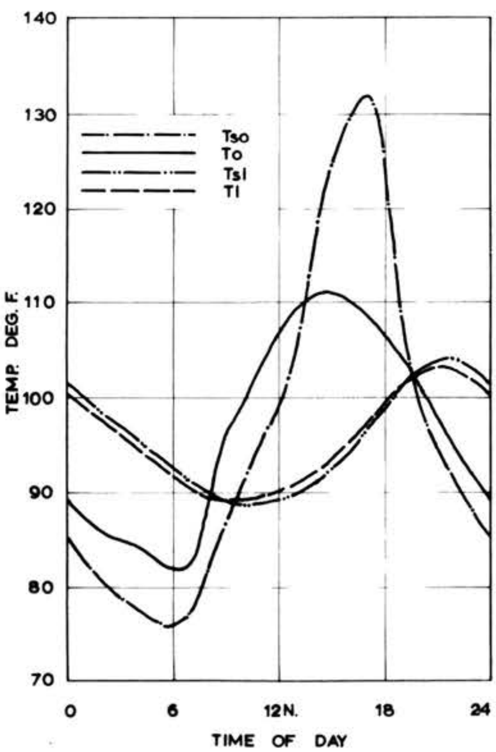


FIG. 11. WEST FACING H. BLOCK WALL VARIABLE INSIDE AIR TEMPERATURE

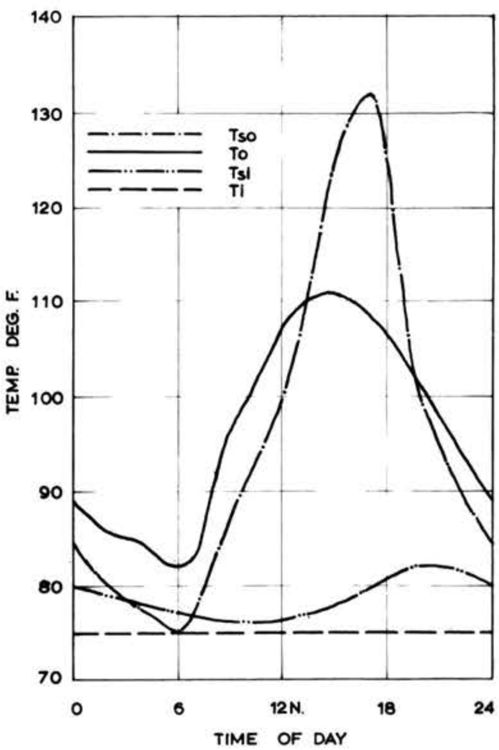


FIG. 12. WEST FACING H. BLOCK WALL CONSTANT INSIDE AIR TEMPERATURE

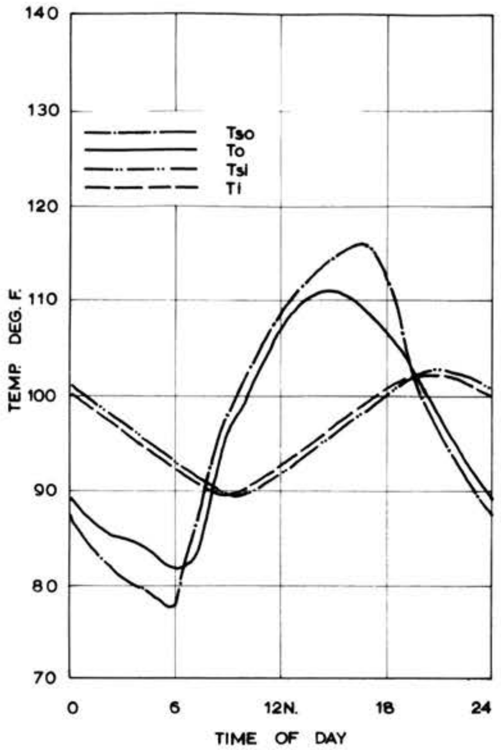


FIG. 13. NORTH FACING H. BLOCK WALL VARIABLE INSIDE AIR TEMPERATURE

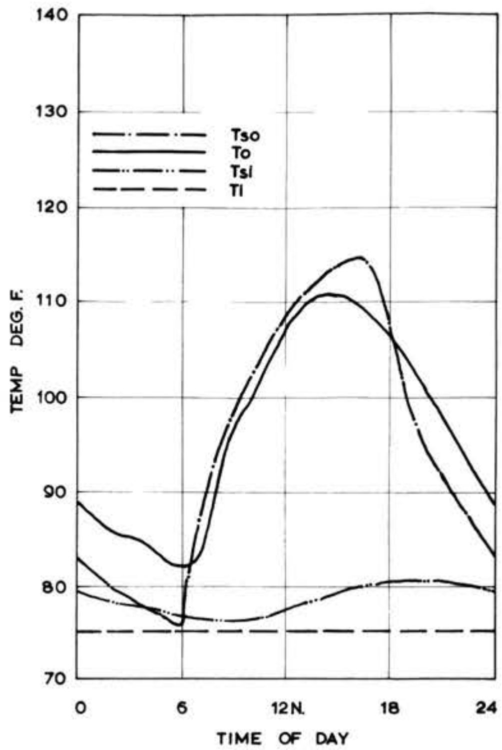


FIG. 14. NORTH FACING H. BLOCK WALL CONSTANT INSIDE AIR TEMPERATURE

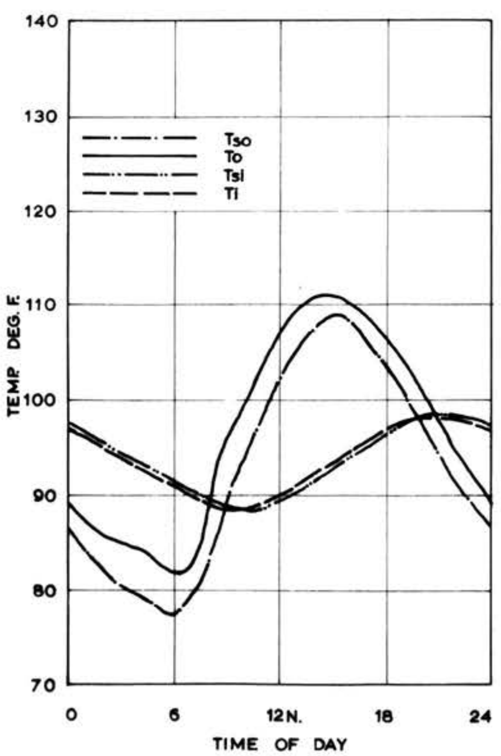


FIG. 15. SOUTH FACING H. BLOCK WALL VARIABLE INSIDE AIR TEMPERATURE

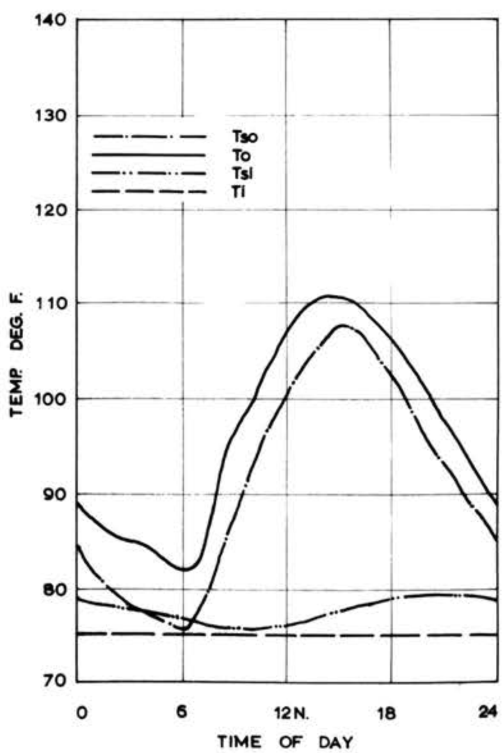


FIG. 16. SOUTH FACING H. BLOCK WALL CONSTANT INSIDE AIR TEMPERATURE

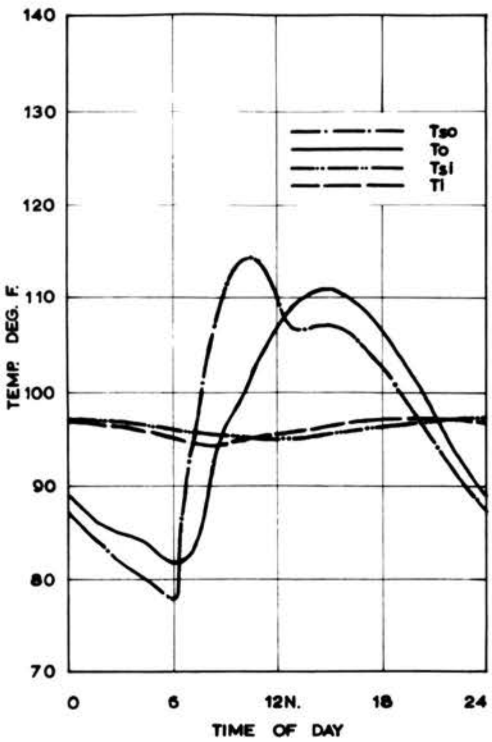


FIG. 17. EAST FACING BRICK WALL VARIABLE INSIDE AIR TEMPERATURE

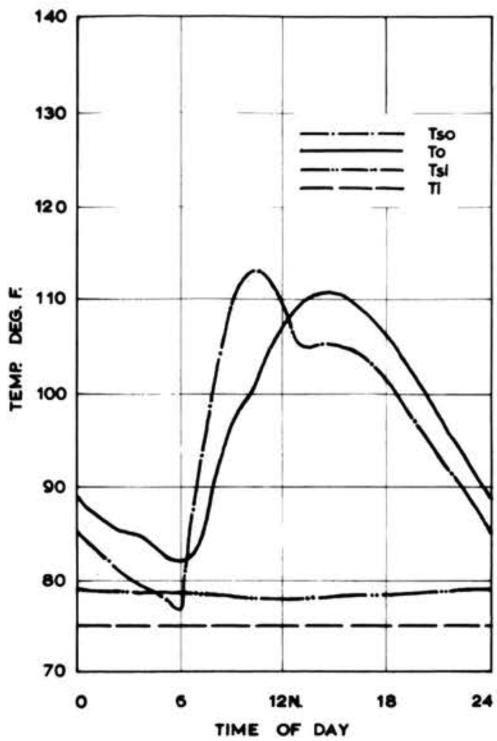


FIG. 18. EAST FACING BRICK WALL CONSTANT INSIDE AIR TEMPERATURE

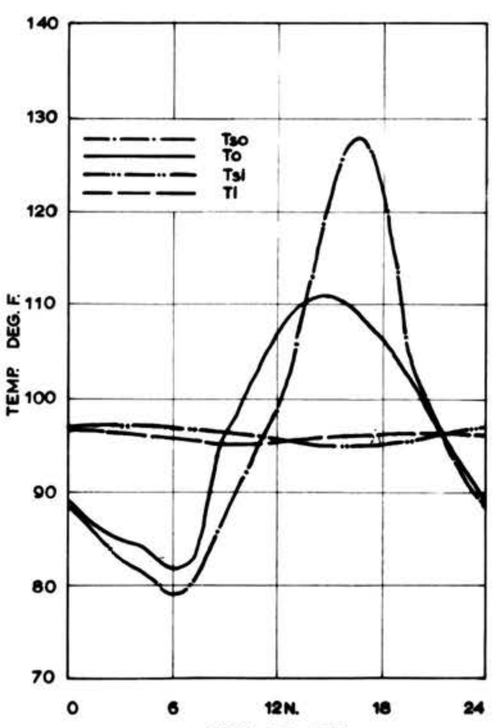


FIG. 19. WEST FACING BRICK WALL VARIABLE INSIDE AIR TEMPERATURE

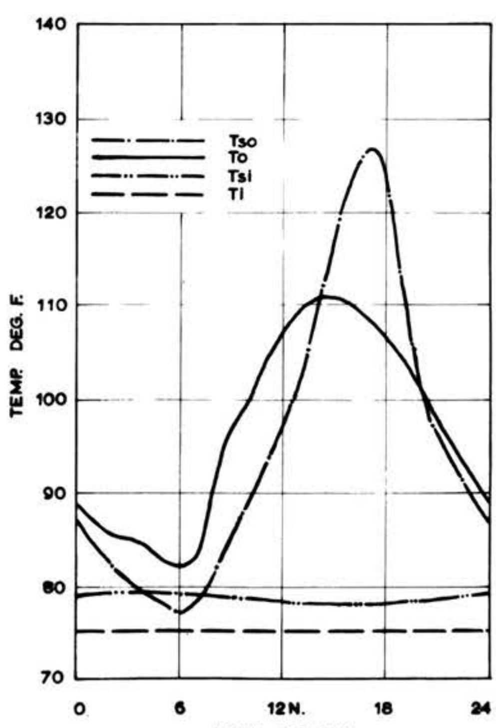


FIG. 20. WEST FACING BRICK WALL CONSTANT INSIDE AIR TEMPERATURE

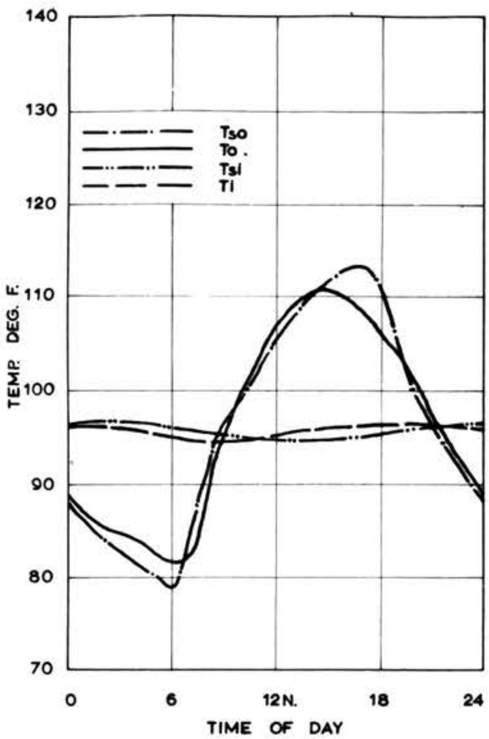


FIG. 21. NORTH FACING BRICK WALL VARIABLE INSIDE AIR TEMPERATURE

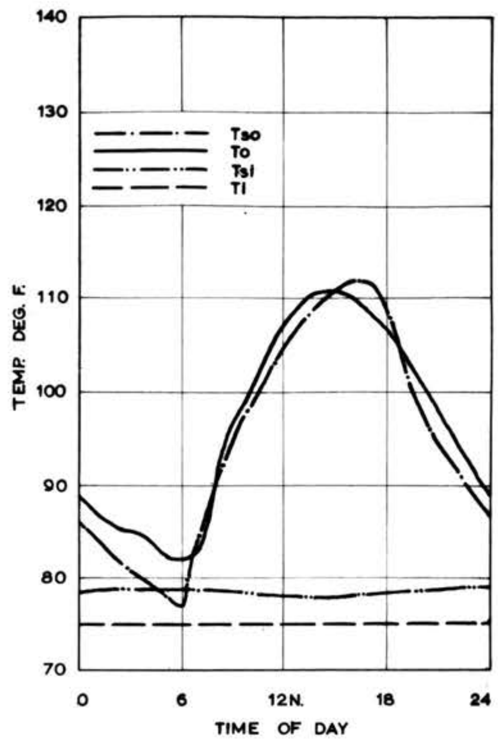


FIG. 22. NORTH FACING BRICK WALL CONSTANT INSIDE AIR TEMPERATURE

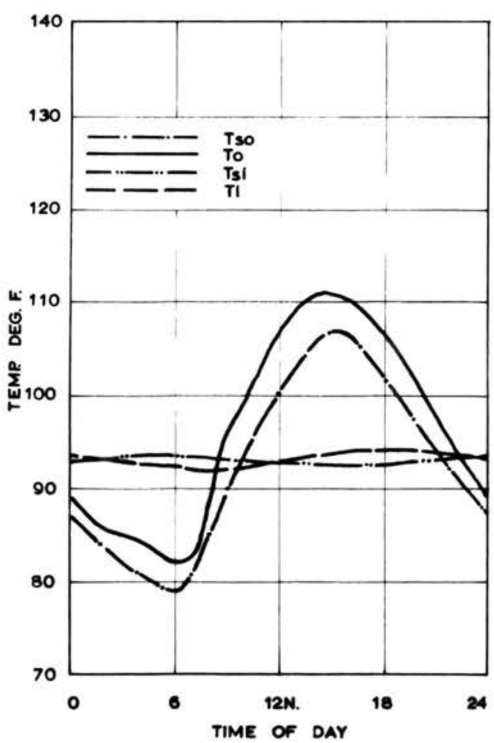


FIG. 23. SOUTH FACING BRICK WALL VARIABLE INSIDE AIR TEMPERATURE

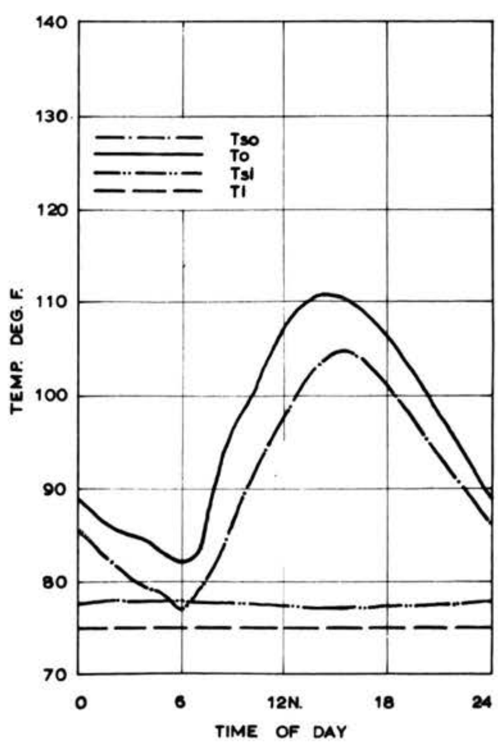


FIG. 24. SOUTH FACING BRICK WALL CONSTANT INSIDE AIR TEMPERATURE

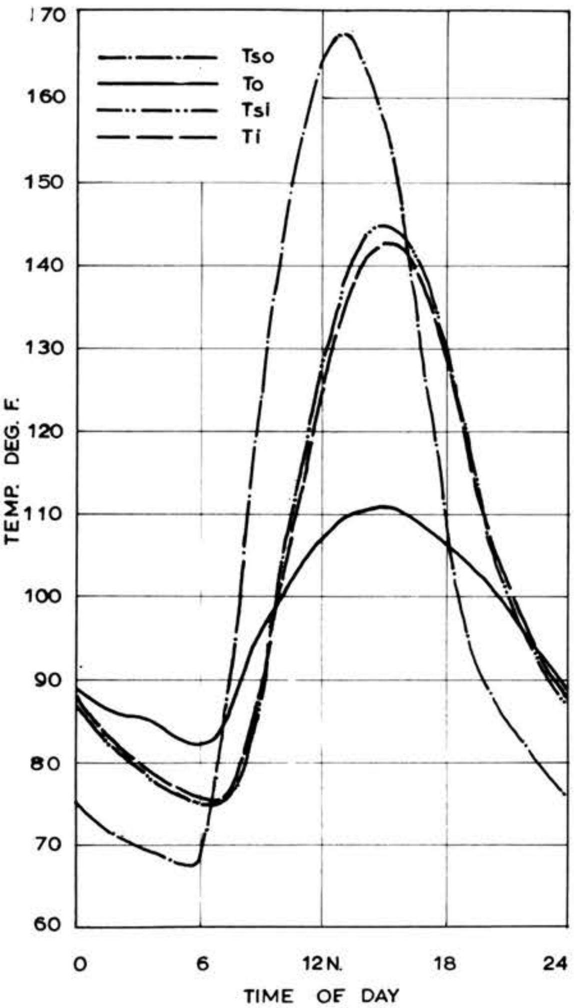


FIG. 25. UNSHADED TIMBER ROOF
VARIABLE INSIDE AIR TEMPERATURE

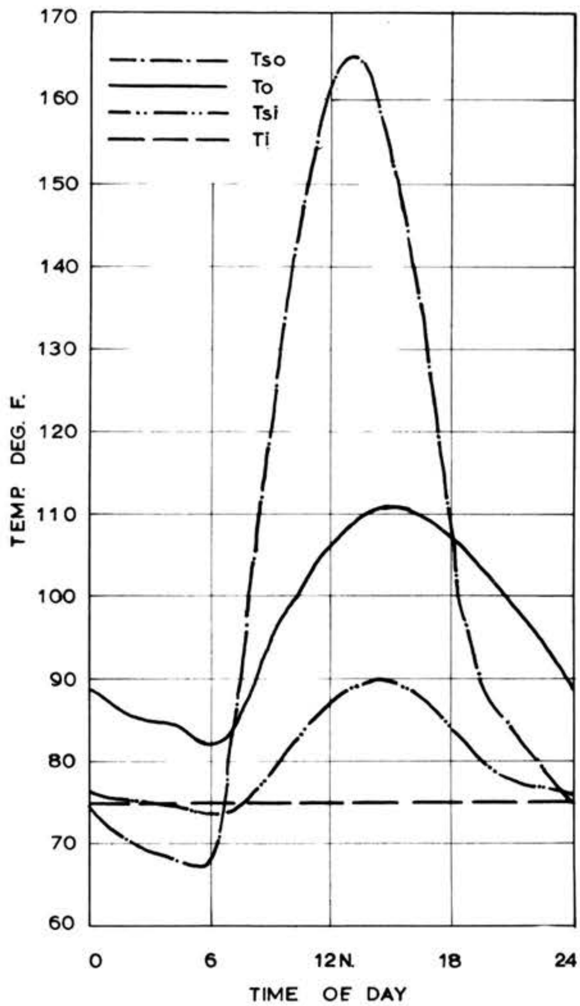


FIG. 26. UNSHADED TIMBER ROOF
CONSTANT INSIDE AIR TEMPERATURE

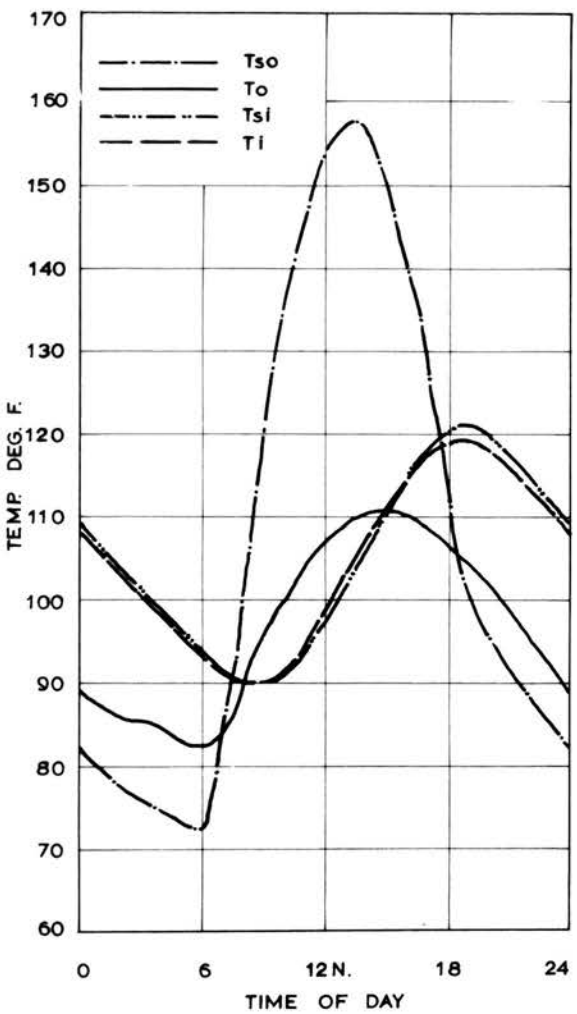


FIG. 27. UNSHADED H. CONCRETE ROOF VARIABLE INSIDE AIR TEMPERATURE

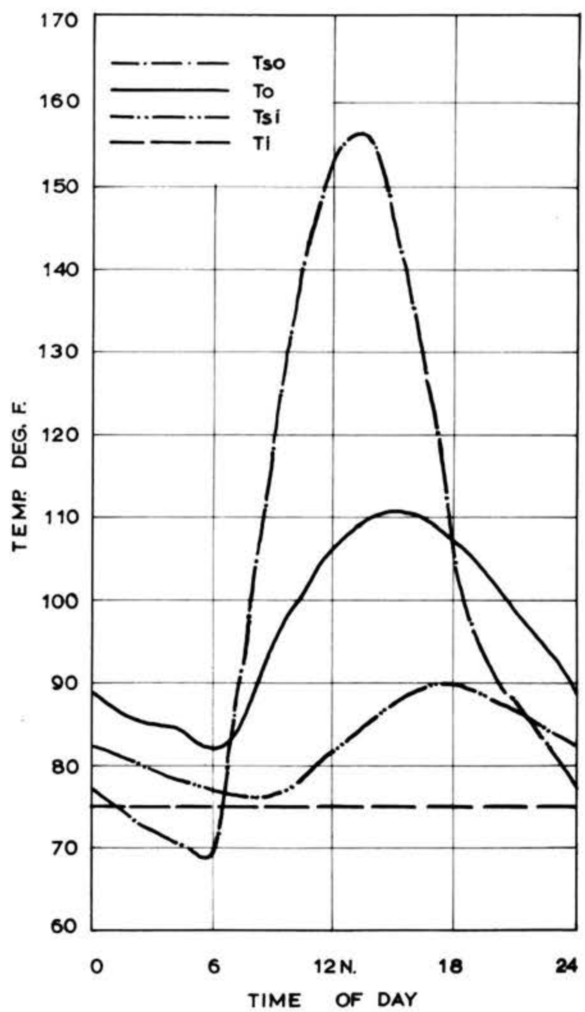


FIG. 28. UNSHADED H. CONCRETE ROOF CONSTANT INSIDE AIR TEMPERATURE

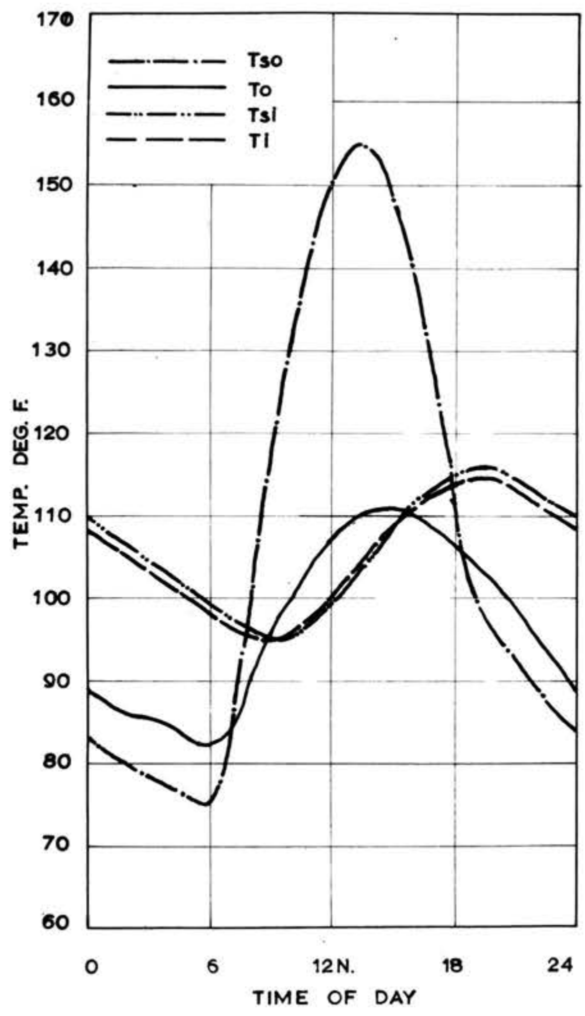


FIG. 29. UNSHADED S. CONCRETE ROOF VARIABLE INSIDE AIR TEMPERATURE

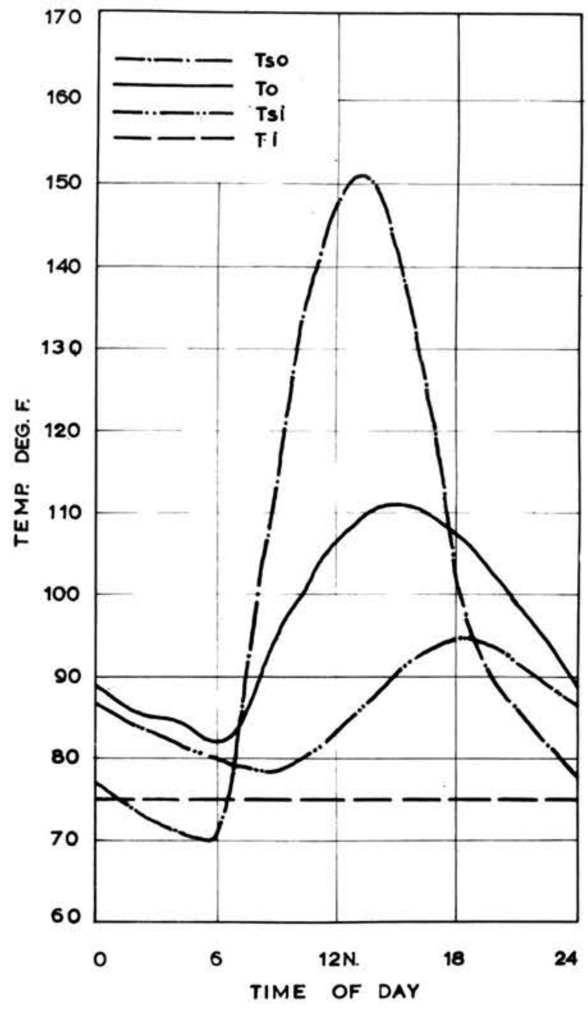


FIG. 30. UNSHADED S. CONCRETE ROOF CONSTANT INSIDE AIR TEMPERATURE

Table 5.1.

Maximum and minimum outside and inside surface temperatures, ^{o}F, and hour of occurrence local time, for uninsulated walls and roofs.

Indoor air temperature variable.

Element	Orientation	Fig.	Outside Surface				Inside Surface			
			Max.	Hour	Min.	Hour	Max.	Hour	Min.	Hour
Timber Wall	East	1	122	10.00	75	06.00	107	17.00	82	07.30
	West	3	137	17.00	74.5	06.00	117	19.00	81.5	08.30
	North	5	118	16.00	75.5	06.00	110	18.30	82	07.30
	South	7	110	15.00	76	06.00	104.5	18.30	81	08.00
Hollow block wall	East	9	118	10.00	76	06.00	102	20.00	89	08.00
	West	11	132	17.00	76	06.00	104	22.00	89	10.00
	North	13	116	16.00	77	06.00	103	21.00	89.5	09.00
	South	15	109	15.00	77	06.00	98.5	21.00	88	10.00
Brick wall	East	17	114	10.00	78	06.00	97	24.00	95	12.00
	West	19	128	17.00	79	06.00	97	03.00	94.5	16.00
	North	21	114	16.00	79	06.00	97	02.00	95	14.00
	South	23	107	15.00	79	06.00	93.5	02.00	92.5	14.00
Timber roof	Horizontal	25	168	13.00	68	06.00	144	15.00	75	07.00
Hollow trough roof	Horizontal	27	158	13.00	73	06.00	121	18.30	90	08.30
Solid concrete roof	Horizontal	29	155	13.00	75	06.00	116	19.00	95	09.00

Table 5.2.

Maximum and minimum outside and inside surface temperatures, ^{o}F, and hour of occurrence local time, for uninsulated walls and roofs.

Indoor air temperature constant = $75^{o}F$.

Element	Orientation	Fig.	Outside Surface				Inside Surface			
			Max.	Hour	Min.	Hour	Max.	Hour	Min.	Hour
Timber Wall	East	2	122	10.00	74.5	06.00	81	11.30	75	06.00
	West	4	136	17.00	74.5	06.00	83.5	18.00	75	08.00
	North	6	117	16.00	74.5	06.00	81	17.30	75	07.00
	South	8	109.5	15.00	74.5	06.00	80	17.00	75	07.00
Hollow block wall	East	10	117	10.00	74.5	06.00	80	17.00	76	08.00
	West	12	132	17.00	75	06.00	82	21.00	76	10.00
	North	14	115	16.00	75.5	06.00	80.5	20.00	76	09.00
	South	16	108	15.00	75.5	06.00	79.5	20.00	75.5	09.00
Brick wall	East	18	113	10.00	77	06.00	79	24.00	78	13.00
	West	20	127	17.00	77	06.00	79.5	03.00	78	16.00
	North	22	112	16.00	77	06.00	79	02.00	78	14.00
	South	24	105	15.00	77	06.00	78	02.00	77	14.00
Timber roof	Horizontal	26	165	13.00	68	06.00	90	14.30	74	07.00
Hollow trough roof	Horizontal	28	156	13.00	69	06.00	90	17.30	76	08.30
Solid concrete roof	Horizontal	30	151	13.00	70	06.00	94	18.00	78	09.00

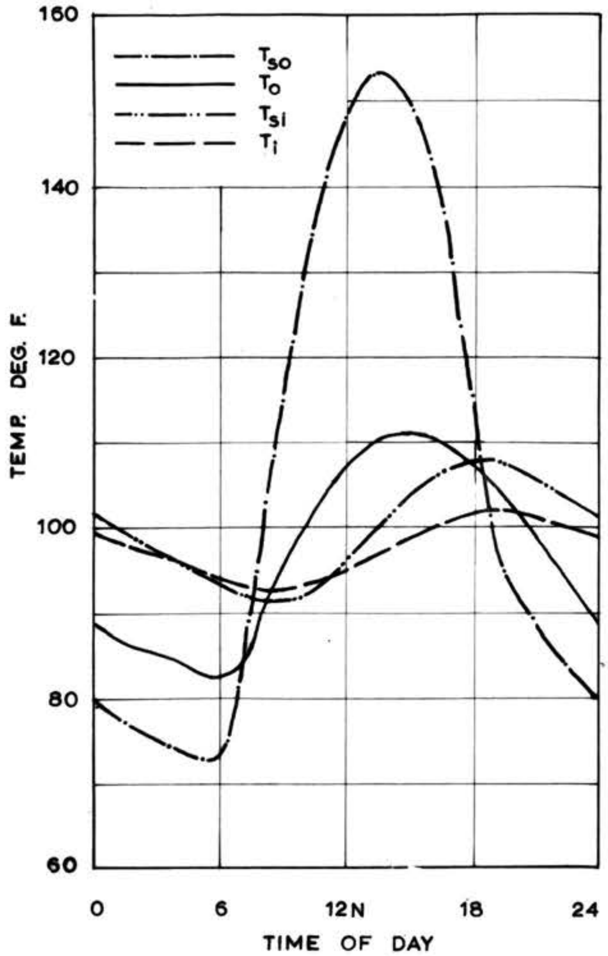


FIG. 31. UNSHADED ENCLOSURE ROOF VARIABLE INSIDE AIR TEMPERATURE

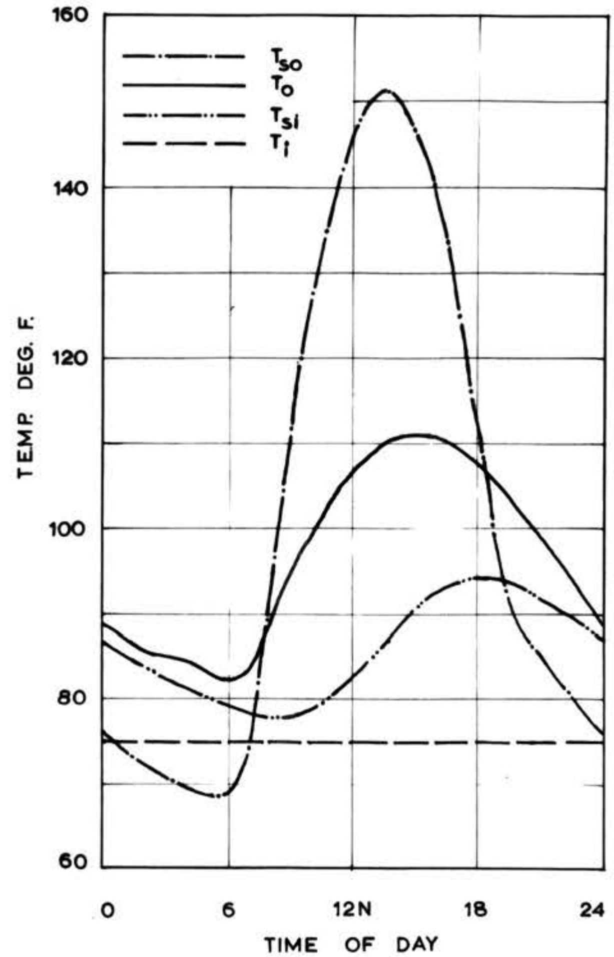


FIG. 32. UNSHADED ENCLOSURE ROOF CONSTANT INSIDE AIR TEMPERATURE

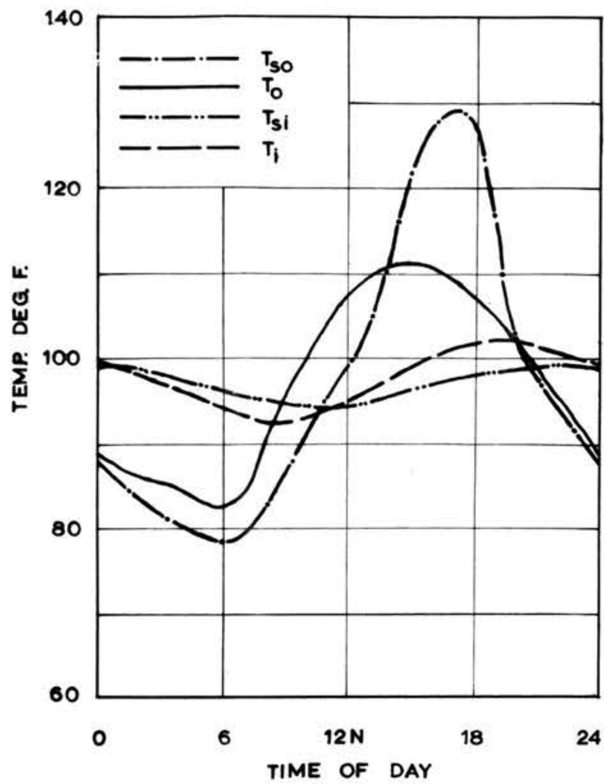


FIG. 33. WEST WALL OF ENCLOSURE
VARIABLE INSIDE AIR TEMPERATURE

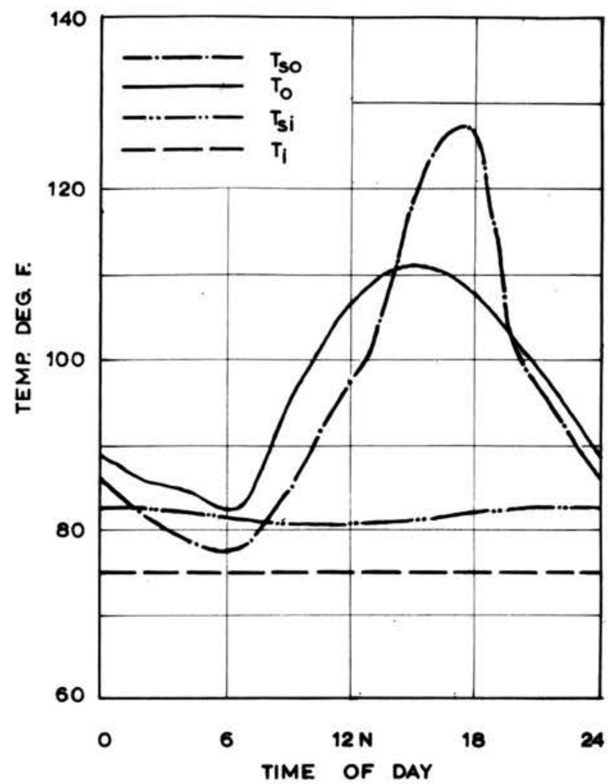


FIG. 34. WEST WALL OF ENCLOSURE
CONSTANT INSIDE AIR TEMPERATURE

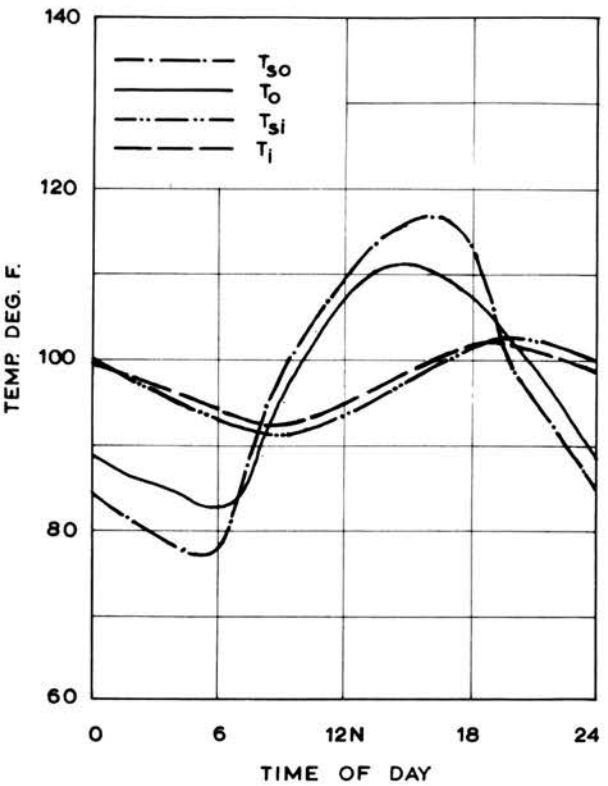


FIG. 35. NORTH WALL OF ENCLOSURE VARIABLE INSIDE AIR TEMPERATURE

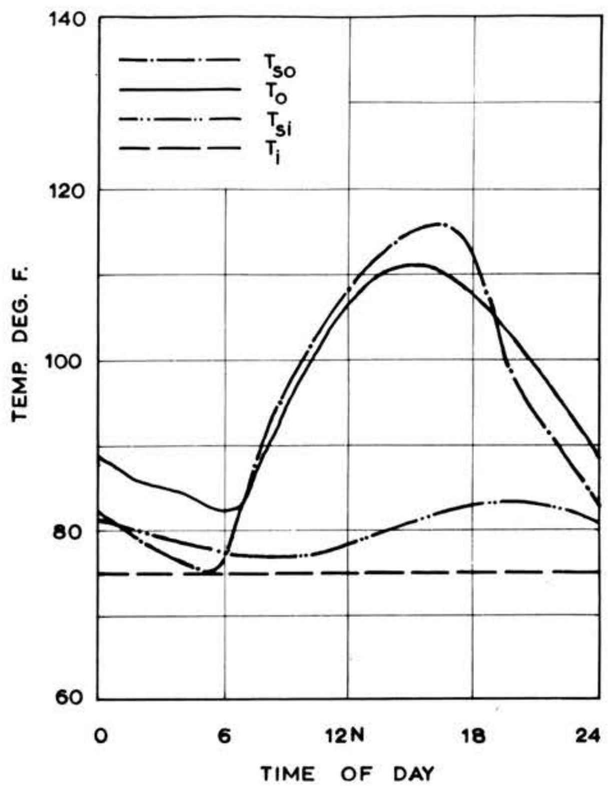


FIG. 36. NORTH WALL OF ENCLOSURE CONSTANT INSIDE AIR TEMPERATURE

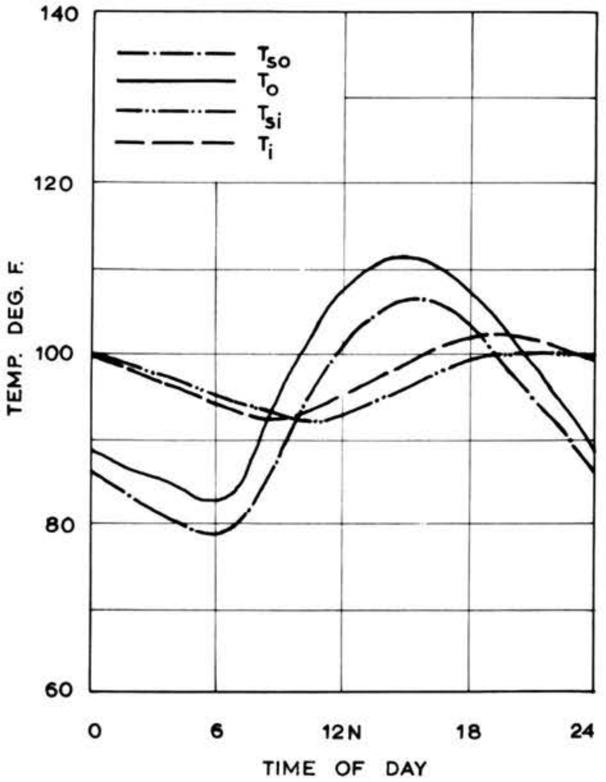


FIG. 37. SOUTH WALL OF ENCLOSURE
VARIABLE INSIDE AIR TEMPERATURE

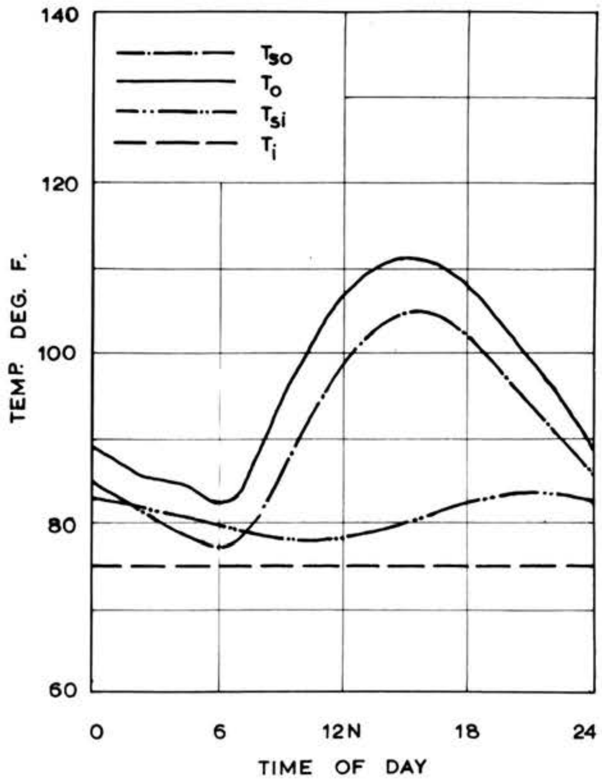


FIG. 38. SOUTH WALL OF ENCLOSURE
CONSTANT INSIDE AIR TEMPERATURE

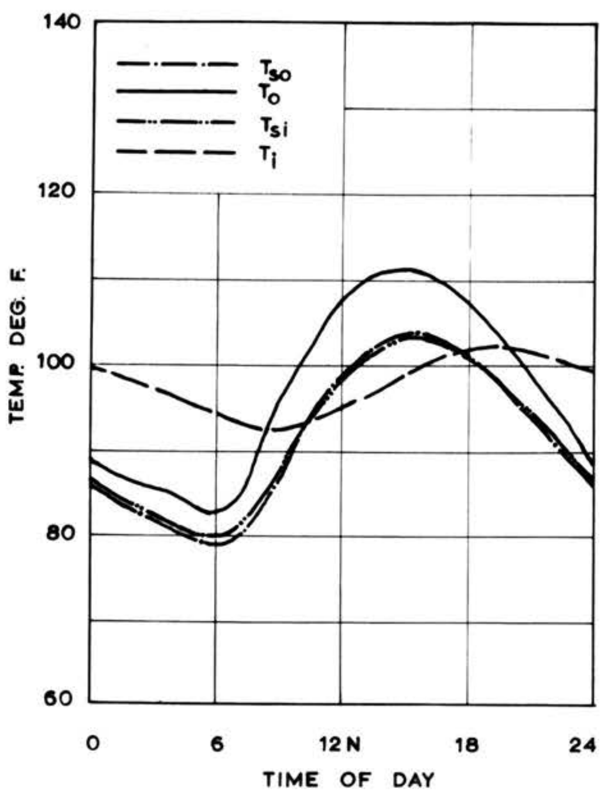


FIG. 39. NORTH & SOUTH WINDOW GLASS VARIABLE INSIDE AIR TEMPERATURE

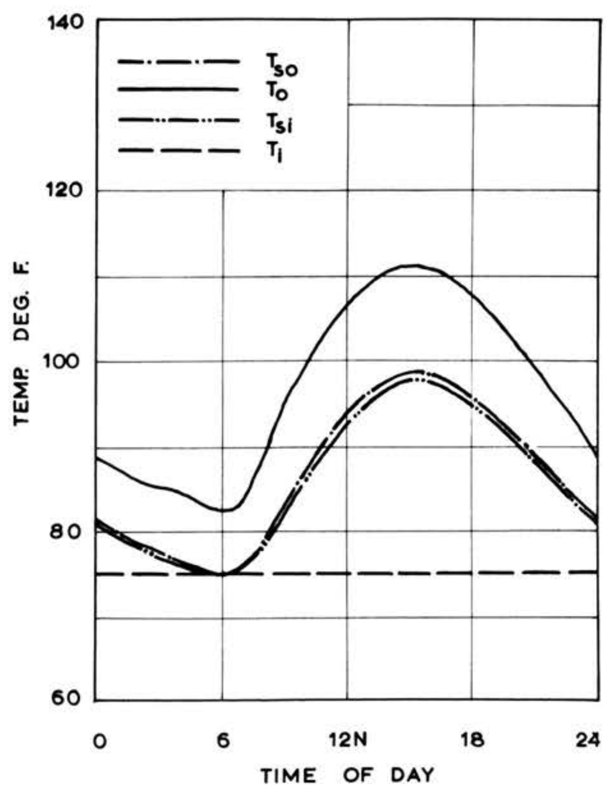


FIG. 40. NORTH & SOUTH WINDOW GLASS CONSTANT INSIDE AIR TEMPERATURE

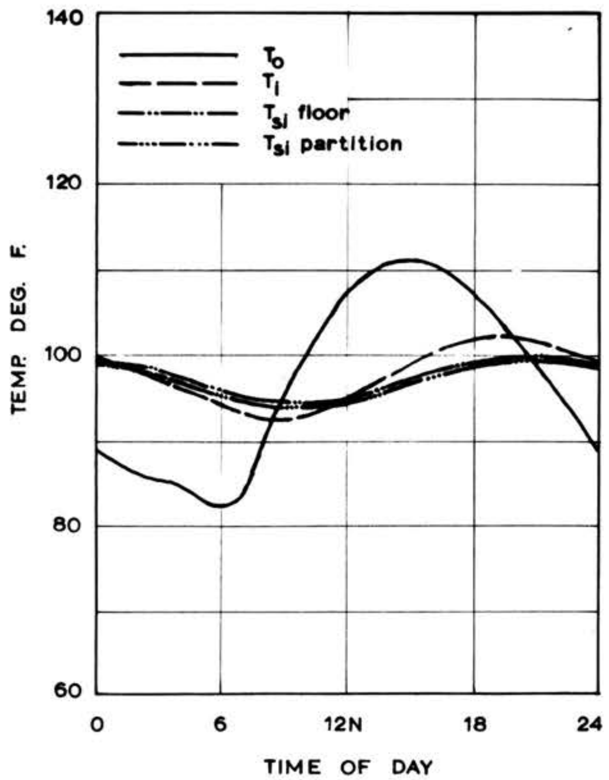


FIG. 41. ENCLOSURE FLOOR & PARTITION
VARIABLE INSIDE AIR TEMPERATURE

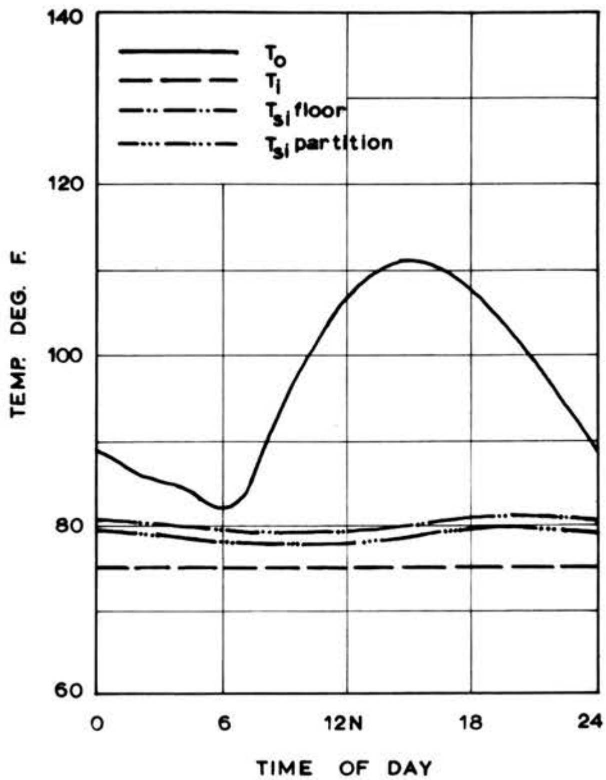


FIG. 42. ENCLOSURE FLOOR & PARTITION
CONSTANT INSIDE AIR TEMPERATURE

Table 5.3.

Maximum and minimum outside and inside surface temperatures, °F, and hour of occurrence local time, for uninsulated enclosure.

Indoor air temperature variable

Max. 102°F at 19.00 Hours

Min. 92°F at 08.30 Hours

Element	Fig.	Outside Surface				Inside Surface			
		Max.	Hour	Min.	Hour	Max.	Hour	Min.	Hour
Roof	31	153	13.00	72	06.00	108	18.00	91	08.30
West Wall	33	129	17.00	79	06.00	99	22.00	95	11.00
North Wall	35	117	16.00	77	06.00	103	20.00	91	09.00
South Wall	37	107	15.00	79	06.00	100	21.00	92	10.30
North Glass	39	104	15.00	79	06.00	103.5	15.00	80	06.00
South Glass	39	104	15.00	79	06.00	103.5	15.00	80	06.00
Floor	41	-	-	-	-	100	20.00	94.5	10.00
Partition	41	-	-	-	-	99.5	20.00	94	10.00

Table 5.4.

Maximum and minimum outside and inside surface temperatures, ^{o}F, and hour of occurrence local time, for uninsulated enclosure.

Indoor air temperature constant = $75^{o}F$

Element	Fig.	Outside Surface				Inside Surface			
		Max.	Hour	Min.	Hour	Max.	Hour	Min.	Hour
Roof	32	151	13.00	69	06.00	94	18.00	78	08.30
West Wall	34	127	17.00	77	06.00	82.5	22.00	80.5	11.00
North Wall	36	116	16.00	75	06.00	83.5	20.00	77	09.00
South Wall	38	105	15.00	77	06.00	83	21.00	78	10.30
North Glass	40	99	15.00	75	06.00	98	15.00	75	06.00
South Glass	40	99	15.00	75	06.00	98	15.00	75	06.00
Floor	42	-	-	-	-	81	20.00	79	10.00
Partition	42	-	-	-	-	80	20.00	78	10.00

5.1.1.1. Outside surface temperatures.

Reference to Tables 5.1 and 5.2, and Figs. (1-30) reveals the following features:

The highest maximum outside surface temperatures occur on roofs. Of these the timber roof has the highest maximum and the largest diurnal variation in outside surface temperature followed by the hollow trough roof while the solid concrete roof has the lowest maximum outside surface temperature and the smallest diurnal variation.

For each wall, the highest maximum outside surface temperature occurs on the west orientation, then the east orientation, then the north orientation, while the south orientation has the lowest maximum.

For each orientation, the highest maximum outside surface temperature and the largest diurnal variation occurs on the timber wall, then the hollow block wall, while the brick wall shows the lowest maximum outside surface temperature and the smallest diurnal variation.

Tables 5.3. and 5.4., and Figs. (31-40), for the enclosure, show that the highest maximum outside surface temperature and the largest diurnal variation occurs on the roof, followed by the west wall, then the north wall (the east wall is a partition), then the south wall, while the north and south window glass shows the lowest maximum and the smallest diurnal fluctuation.

For all cases (Figs. 1-40) the outside surface temperature drops below the outdoor air temperature during the early hours of the day, before sunrise, and, except for south facing walls and north and south window glass, reaches values higher than the outdoor air temperature sometime during the day. (South facing walls and north and south window glass are in shade all day).

All outside surface temperature minima occur, just before sunrise, at about 6.00 hours local time. The outside surface temperature maxima, for all roofs, seem to occur at about 13.00 hours local time. East facing walls reach their maxima earliest at about 10.00 hours local time, followed by south facing walls at about 15.00 hours local time, then north facing walls at about 16.00 hours local time, while west facing walls reach their outside surface temperature maxima latest at about 17.00 hours local time.

The outside surface temperature maxima and minima in cases where the indoor air temperature is assumed constant (Tables 5.2. and 5.4.) are usually a few degrees lower than the maxima and minima for corresponding cases (Tables 5.1 and 5.3) where the indoor air temperature is assumed variable.

5.1.1.2. Inside surface temperatures.

Referring to Tables 5.1. and 5.2. it will be seen that, for each wall, the highest maximum inside surface temperature occurs when the wall faces west, while the lowest maximum occurs when the wall faces south. Maxima, when the wall faces east or north, lie in between and are sometimes equal, and in the case of the brick wall with variable indoor air temperature both are equal to the maximum attained on the west orientation.

For each orientation, the timber wall attains the highest maximum inside surface temperature and has the largest diurnal variation, followed by the hollow block wall, while the brick wall has the lowest maximum and the smallest diurnal variation.

For each wall, the maximum and minimum inside surface temperatures are reached earliest when the wall faces east, and latest when the wall faces west, while the east and north facing walls reach these temperatures sometime in between, and in most cases at the same time.

For each orientation, the timber wall attains maximum and minimum inside surface temperatures earliest, followed by the hollow block wall, while the brick wall attains these temperatures latest.

Of the roofs, when the indoor temperature is variable (Table 5.1.), the timber roof reaches the highest maximum inside surface temperature, and has the largest diurnal variation, followed by the hollow trough roof, while the solid concrete roof has the lowest maximum (lower even than west facing timber wall) and the smallest diurnal variation.

However, when the indoor air temperature is assumed constant (Table 5.2.), the solid concrete roof attains the highest inside surface temperature maximum and minimum, and has the same diurnal variation as the timber roof, which attains lower maximum and minimum temperatures. The hollow trough roof reaches the same inside surface temperature maximum as the timber roof, but has the smallest diurnal inside surface temperature variation of the three roofs.

The maximum and minimum temperatures are, however, always reached earliest by the timber roof, followed by the hollow trough roof, and are reached latest by the solid concrete roof.

Tables 5.3 and 5.4. show that the largest diurnal variation of inside surface temperature, in the enclosure, occurs on the north and south window glass. Inside surface temperature maxima and minima show no appreciable lag from the times of occurence of maxima and minima on the outside surface.

When the indoor air temperature is variable (Table 5.3.) the roof shows the highest inside surface temperature maximum, which, except for the glass, like the minimum occurs earlier than all other surfaces. The west wall, however, has the lowest inside surface temperature maximum, which, like the minimum occurs latest of all, and the smallest diurnal variation.

When the indoor air temperature is assumed constant (Table 5.4.), glass attains the highest maximum inside surface temperature, followed by the roof which, except for glass, attains its maximum and minimum temperatures earlier than other surfaces. The west wall which has the smallest diurnal temperature variation, attains its maximum and minimum temperatures latest of all enclosure surfaces.

In all the cases reported, Figs. (1-42), the inside surface temperature maxima and minima are noticeably lower for cases where the indoor temperature is assumed constant at 75°F. (Tables 5.2. and 5.4.) than the corresponding cases (Tables 5.1. and 5.3.) where the indoor temperature is assumed variable.

5.1.2. Discussion

A comparison can be made between Figs. 31-32, for the enclosure roof, and Figs. 29-30, for the identical concrete roof in the series on single elements. Also Figs. 33-34, for the west wall of the enclosure, can be compared to Figs. 19-20, for the identical west facing brick wall in the series on single elements, and Figs. 35-36, for the north wall of the enclosure, to Figs. 13-14, for the identical north facing hollow block wall in the series on single elements.

The corresponding curves are fairly similar although the maximum and minimum temperatures are, in some cases, different for the enclosure element. The difference is within 3°F for all curves except those for the roof, when the indoor temperature is assumed variable, where the maximum is 8°F and the minimum 4.5°F, higher for the roof, in the series on single elements, than the enclosure roof.

Also, shorter time lags are observed for the west wall of the enclosure, than for the identical single west facing brick wall. The other enclosure elements, which can be compared to their single equivalents: the roof and north wall, however, show appreciable agreement, where, although the enclosure elements have shorter time lags, the differences are within one hour.

These discrepancies, in temperature range and time lags, found between enclosure and single elements, can probably be attributed to the different ways in which the internal radiation network has been represented in the two cases. A more accurate representation was possible for the enclosure. Also the single elements may have suffered from the lack of internal capacity effects present in the enclosure set-up.

Raychaudhuri, Jain and Yadva[1] compared computed and measured time lags for walls of experimental houses, and indicated that because their method of computation did not take properly into account the internal radiation processes, the computed time-lag property was found to show a discrepancy from the measured property.

It will also be noticed that when the indoor air temperature is assumed variable for single elements, the indoor air temperature curve and the inside surface temperature curve stay close to one another. This is because, in effect, the inside surface is assumed to be surrounded by surfaces at the same temperature. In the enclosure where the indoor air temperature node is coupled to nodes representing other inside surfaces,

1. Raychaudhuri, B.C., Jain, S.P., and Yadva, K.G., "Thermal characteristics of unconditioned insulated masonary buildings in hot arid regions". Int. Journal of Biometeorology, Vol. 8, No.2, 1964.

which are not necessarily at the same temperature, this does not occur and the indoor air temperature curve is distinguishable from those of inside surfaces.

In the enclosure the time lags are observed to be the same for each element whether the enclosure is conditioned or not. For single elements slight differences are sometimes found between cases where the inside air temperature is assumed constant and those when this is assumed to be variable. This may also be attributable to the different representation of the indoor boundary conditions, and the weight given to the inside surface-air coupling.

As indoor boundary conditions are reproduced more accurately for the enclosure, it can safely be assumed that the curves for enclosure elements are more accurate than the curves for single elements. For single elements, a relatively smaller error probably occurs, due to the inadequate allowance for radiation exchange at the inside surface, when the indoor air temperature is assumed constant because of the lower differentials between inside surface temperatures in that case.

The results reported show the thermal importance of roof elements and west facing walls. The influence of shading is evident in that south facing walls, which are in shade, remain cooler than walls otherwise oriented, and maintain an outside surface temperature lower than that of the outdoor air.

The influence of surface finish and colour, has not been investigated in this thesis; but it is well documented that lighter coloured finishes, which lead to a reduction in solar absorptivity, result in lower surface temperatures. Givoni[2], Roux[3], and van Straaten[4], for example, report findings to this effect.

Heavier elements are found to afford a greater modification of the outdoor environment, than lighter elements. It can be seen, however, that in unconditioned structures it is not possible to suppress indoor temperatures to a level compatible with thermal comfort conditions.

Similar evidence was reported by Raychaudhuri, Ali and Garg[5], who investigated temperatures in full scale test houses when, during the summer with outdoor air temperature maxima just over 100°F, they found that in all cases the indoor temperatures were consistantly above the comfort zone taken at 73.5°F, effective temperature. Puchberg[*] reported similar findings in an investigation of summer conditions in buildings in Fresno, U.S.A.

---

2. Givoni, B., and Shalon, R., "Influence of roof type and construction on indoor thermal conditions in Beeer Shiva". Research paper No. 11. Building Research Station, Haifa. 1962.
3. Roux, A.J.A., "Heat interchange between a roof and its surroundings". N.B.R.I. Bulletin No. 1, C.S.I.R., Pretoria, 1948.
4. van Straaten, J.F., "The thermal performance characteristics of curtain wall construction in warm climates". R.D. 33. C.S.I.R., Pretoria, 1961.
5. Raychaudhuri, B.C., Ali, S., and Garg, D.P., "Indoor climate of residential buildings in hot arid regions". Building Science, Vol. 1, Pergamon Press London, 1964.

* loc.cit see footnote 3.15.

5.2. INSULATION STUDIES

5.2.1. Results

The variation of outside and inside surface temperatures with increments of insulation is shown in Tables 5.5 - 5.8 for the timber wall, Tables 5.9 - 5.12 for the hollow block wall, and Tables 5.13 - 5.16 for the brick wall. One Table is given for each of the four orientations, east, west, north and south. Table 5.17 gives the surface temperatures for the timber roof, Table 5.18 for the hollow trough roof, and Table 5.19 for the solid concrete roof.

Each Table gives surface temperatures for the insulation placed on the outside and the inside surfaces (as shown in Figs 1-6 and 9-11 of the previous chapter), alternatively. For the timber wall, Tables 5.5 - 5.8, no difference was found in surface temperatures when the insulation is placed on either side of the cavity; so results are shown with the insulation on the inside surface only.

Curves showing the variation of inside surface temperatures with increments of outside and inside insulation are given in Figs 43 and 44 for the west (and south) facing timber wall, Figs 45 and 46 for the west facing hollow block wall, Figs 47 and 48 for the west facing brick wall, Figs 49 and 50 for the timber roof, Figs 51 and 52 for the hollow trough roof, and Figs 53 and 54 for the solid concrete roof.

The curves for the different orientations of each wall type are similar in pattern varying only in temperature ranges (as can be seen from Tables 5.5 - 5.16). It was, therefore, thought adequate to present the figures for the west orientation, to bring out the relevant features.

The surface temperatures of the elements of the insulated enclosure

(See section 4.3.3.) are given in Table 5.20.

Since insulation has been assumed of negligible heat capacity, the times of occurrence of maximum and minimum temperatures for insulated elements are the same as those given for the uninsulated elements in Tables 5.1 - 5.4.

5.2.2. Effect of location of insulation on surface temperature.

5.2.2.1. Outside surface temperatures.

Reference to Tables 5.5 - 5.20 shows that higher outside surface temperatures are attained when insulation is placed on the outside surface, than on the inside surface of an element. When insulation is placed against the inside surface the outside surface temperature range seems to be only slightly, or not at all, affected. With outside insulation, however, the outside surface temperature range increases sharply with the first increment of insulation and then more slowly as insulation is increased until it is not further affected.

Similarly oriented elements tend to reach more or less the same outside surface temperature maxima and minima as insulation resistance approaches 10 $^{\circ}$Fhr.ft.^{2}/Btu, the equivalent of 2.5 inches of polystyrene. This range for roofs is about 67-172 $^{\circ}$F, for west facing walls about 74-140 $^{\circ}$F, for east facing walls about 74-124 $^{\circ}$F, for north facing walls about 75-119 $^{\circ}$F, and for south facing walls about 75-111 $^{\circ}$F.

When indoor air temperature is assumed constant the outside surface temperature maxima and minima tend to be slightly lower than when the indoor air temperature is assumed variable, for otherwise identical cases.

Table 5.5.

EAST FACING TIMBER WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0					75	122	82	107
2					74.5	122	83.5	105.5
4					74.5	122	85	104
6					-	-	-	-
8					74.5	122	86.5	102
10					-	-	-	-
12					74.5	122	87.5	101.5
	Indoor air temperature constant = 75°F							
0					74.5	122	75	81
2					74	122	75	79
4					74	122	75	78
6					-	-	-	-
8					74	122	75	77
10					-	-	-	-
12					74	122	75	76.5

Table 5.6.

WEST FACING TIMBER WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, ^{o}F.

Insulation Resistance $^{o}F.hr.ft.^{2}$ /Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0					74.5	137	81.5	117
2					74.5	137.5	83.5	111.5
4					74.5	137.5	85	108.5
6					-	-	-	-
8					74.5	137.5	86.5	105.5
10					-	-	-	-
12					74.5	137.5	87.5	104
	Indoor air temperature constant = $75^{o}F$							
0					74.5	136	75	83.5
2					74	137	75	80.5
4					74	137	75	79
6					-	-	-	-
8					74	137	75	77.5
10					-	-	-	-
12					74	137	75	77

Table 5.7.

NORTH FACING TIMBER WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0					75.5	118	82	110
2					75	119	84	108
4					75	119	85.5	106
6					-	-	-	-
8					75	119	86.5	104
10					-	-	-	-
12					75	119	87.5	102.5
	Indoor air temperature constant = 75°F							
0					74.5	117	75	81
2					74.5	118	75	79
4					74.5	118	75	78
6					-	-	-	-
8					74.5	118	75	77
10					-	-	-	-
12					75	118	75	76.5

Table 5.8.

SOUTH FACING TIMBER WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.² /Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0					76	110	81	104.5
2					75.5	110.5	83	103
4					75.5	111	84	102
6					-	-	-	-
8					75	111	85.5	100
10					-	-	-	-
12					75	111	86.5	99.5
	Indoor air temperature constant = 75°F							
0					74.5	109.5	75	80
2					75	110	75	78
4					75	110	75	77.5
6					-	-	-	-
8					75	110.5	75	76.5
10					-	-	-	-
12					75	111	75	76

Table 5.9.

EAST FACING HOLLOW BLOCK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, ^{o}F.

Insulation Resistance $^{o}F.hr.ft.^2/Btu$	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	76	118	89	102	76	118	89	102
2	74.5	123	91.5	98.5	76	118.5	89.5	101.5
4	74	123.5	92.5	98	76	119	89.5	101.5
6	-	-	-	-	-	-	-	-
8	74	124	93	97.5	76	119	90	101
10	-	-	-	-	-	-	-	-
12	74	127	93	97.5	76	119	90	100.5
	Indoor air temperature constant = $75^{o}F$							
0	74.5	117	76	80	74.5	117	76	80
2	74	112	75.5	77.5	75	118	75.5	77.5
4	73.5	123	75.5	76.5	75	118	75.5	77
6	-	-	-	-	-	-	-	-
8	73.5	123.5	75	76	75.5	118	75	76
10	-	-	-	-	-	-	-	-
12	73.5	123.5	75	75.5	75.5	118	75	75.5

Table 5.10.

WEST FACING HOLLOW BLOCK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	76	132	89	104	76	132	89	104
2	75	137.5	92.5	100.5	76	133.5	89.5	103
4	74.5	138.5	93	93.5	76.5	134	90	102
6	-	-	-	-	-	-	-	-
8	74	139	93.5	98	76.5	134	90.5	101
10	-	-	-	-	-	-	-	-
12	74	139	93.5	98	76.5	134	90.5	100.5
	Indoor air temperature constant = 75°F							
0	75	132	76	82	75	132	76	82
2	74	137	75.5	78.5	75.5	133.5	75.5	78.5
4	73.5	138	75.5	77.5	75.5	133.5	75.5	77.5
6	-	-	-	-	-	-	-	-
8	73.5	138.5	75	76.5	75.5	133.5	75	76.5
10	-	-	-	-	-	-	-	-
12	73.5	138.5	75	76	75.5	133.5	75	76

Table 5.11.

NORTH FACING HOLLOW BLOCK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.^{2}/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	77	116	89.5	103	77	116	89.5	103
2	75.5	118.5	92	99	77	117	89.5	102
4	75	118.5	93	98	77	117	90	101.5
6	-	-	-	-	-	-	-	-
8	75	119	93	97.5	77	117	90	100.5
10	-	-	-	-	-	-	-	-
12	75	119	93	97.5	77	117	90.5	100
	Indoor air temperature constant = 75°F							
0	75.5	115	76	80.5	75.5	115	76	80.5
2	74.5	118	75.5	78	76	117	75.5	78
4	74.5	118.5	75.5	77	76.5	117	75.5	77.5
6	-	-	-	-	-	-	-	-
8	74.5	119	75	76	76.5	117	75	76
10	-	-	-	-	-	-	-	-
12	74.5	119	75	76	76.5	117	75	76

Table 5.12.

SOUTH FACING HOLLOW BLOCK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	77	109	88	98.5	77	109	88	98.5
2	75	111	90.5	96	77	109	88	98.5
4	75	111.5	91	95.5	77	109	88.5	98
6	-	-	-	-	-	-	-	-
8	75	111.5	91.5	95.5	77	109	89	98
10	-	-	-	-	-	-	-	-
12	75	111.5	91.5	95.5	77	109	89	98
	Indoor air temperature constant = 75°F							
0	75.5	108	75.5	79.5	75.5	108	75.5	79.5
2	75	110.5	75.5	77.5	76	108.5	75.5	77.5
4	74.5	111	75.5	76.5	76.5	108.5	75.5	76.5
6	-	-	-	-	-	-	-	-
8	74.5	111	75	76	76.5	108.5	75	76
10	-	-	-	-	-	-	-	-
12	74.5	111	75	76	76.5	108.5	75	76

Table 5.13.

EAST FACING BRICK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	78	114	95	97	78	114	95	97
2	75	122	95	96.5	78.5	114.5	94	97.5
4	74	124	95	96	78.5	114.5	93	98
6	74	124	95	96	78.5	115	92.5	98.5
8	-	-	-	-	-	-	-	-
10	74	124	95	96	78.5	115	92.5	98.5
12	-	-	-	-	-	-	-	-
	Indoor air temperature constant = 75°F							
0	77	113	78	79	77	113	78	79
2	74	121	77	77.5	77.5	114	77	77.5
4	73.5	123	76.5	76.5	78	114.5	76.5	77
6	73.5	123	76.5	76.5	78	115	76.5	76.5
8	-	-	-	-	-	-	-	-
10	73.5	123	76	76	78	115	76	76
12	-	-	-	-	-	-	-	-

Table 5.14.

WEST FACING BRICK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	79	128	94.5	97	79	128	94.5	97
2	75.5	137	95	96.5	78.5	128	93.5	97.5
4	74.5	139	95	96	78.5	128.5	93	98
6	74	139.5	95	96	78.5	128.5	92	98.5
8	-	-	-	-	-	-	-	-
10	74	140	95	96	78.5	129	92	98.5
12	-	-	-	-	-	-	-	-
	Indoor air temperature constant = 75°F							
0	77	127	78	79.5	77	127	78	79.5
2	74.5	136.5	77	78	77.5	127	76.5	77.5
4	74	138	76	77	77.5	127	76	77
6	73.5	139	76	76.5	78	128	76	76.5
8	-	-	-	-	-	-	-	-
10	73.5	139	76	76	78.5	128	76	76
12	-	-	-	-	-	-	-	-

Table 5.15.

NORTH FACING BRICK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	79	114	95	97	79	114	95	97
2	76	117	95	96.5	79	114.5	94	97.5
4	75	118	95	96	79	114.5	93	98.5
6	75	119	95	96	79	114.5	92.5	98.5
8	-	-	-	-	-	-	-	-
10	75	119	95	96	79	114.5	92	98.5
12	-	-	-	-	-	-	-	-
	Indoor air temperature constant = 75°F							
0	77	112	78	79	77	112	78	79
2	75	116	77	77.5	78	113	77	77.5
4	75	118	76.5	77	78.5	114	76.5	77
6	75	118	76.5	76.5	78.5	114	76.5	76.5
8	-	-	-	-	-	-	-	-
10	75	118	76	76	78.5	114	76	76
12	-	-	-	-	-	-	-	-

Table 5.16.

SOUTH FACING BRICK WALL

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.^{2}/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	79	107	92.5	93.5	77	107	92.5	93.5
2	76.5	110	92.5	93.5	79	107	91.5	95
4	75.5	111	92.5	93.5	79	107	90.5	96
6	75.5	111	92.5	93.5	79	107	90	96.5
8	-	-	-	-	-	-	-	-
10	75.5	111	92.5	93.5	79	107	90	96.5
12	-	-	-	-	-	-	-	-
	Indoor air temperature constant = 75°F							
0	77	105	77	78	77	105	77	78
2	75	109	76.5	77	78	105	76.5	77
4	75	111	76.5	76.5	78	106	76	76.5
6	75	111	76	76	78	106	76	76
8	-	-	-	-	-	-	-	-
10	75	111	76	76	78	106	76	76
12	-	-	-	-	-	-	-	-

Table 5.17.

TIMBER ROOF

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.²/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	68	168	75	144	68	168	75	144
2	68	170	80	131	67.5	169	77	137
4	68	170	83	122	67.5	169	78.5	132
6	-	-	-	-	-	-	-	-
8	67.5	171	86.5	115	67.5	169.5	80	126.5
10	-	-	-	-	-	-	-	-
12	67	172	88.5	111	67.5	170	81.5	120.5
	Indoor air temperature constant = 75°F							
0	68	165	74	90	68	165	74	90
2	67	167.5	74.5	84.5	67.5	167	74.5	84.5
4	67	169	75	82	67.5	169	75	82
6	-	-	-	-	-	-	-	-
8	67	171	75	79.5	67	169	75	79.5
10	-	-	-	-	-	-	-	-
12	67	171	75	78.5	67	169	75	78.5

Table 5.18.

HOLLOW TROUGH ROOF

Variation of minimum and maximum, outside and inside surface temperatures with insulation, ^{o}F.

Insulation Resistance $^{o}F.hr.ft.^{2}$/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	73	158	90	121	73	158	90	121
2	70	167	97	110	72.5	159	91	117
4	69	170	98.5	107	72	159	91.5	114
6	-	-	-	-	-	-	-	-
8	68	171	98.5	104	72	159	92	109.5
10	-	-	-	-	-	-	-	-
12	68	172	98.5	103	72	159	93	108
	Indoor air temperature constant = 75^{o}F							
0	69	156	76	90	69	156	76	90
2	68	166	76	82.5	70.5	157	76	83
4	68	169	76	80	71	158	76	80.5
6	-	-	-	-	-	-	-	-
8	67	171	75.5	78	71	158	75.5	78
10	-	-	-	-	-	-	-	-
12	67	171	75.5	77	71	158	75.5	77

Table 5.19.

SOLID CONCRETE ROOF

Variation of minimum and maximum, outside and inside surface temperatures with insulation, °F.

Insulation Resistance °F.hr.ft.^{2}/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor air temperature variable							
0	75	155	95	116	75	155	95	116
2	70	167	100.5	107.5	75	155	95	113
4	69	169	101	105	75	155	95	111
6	-	-	-	-	-	-	-	-
8	68	171	100	103	75	155	95	107.5
10	-	-	-	-	-	-	-	-
12	67	172	99.5	102	75	155	95	106
	Indoor air temperature constant = 75°F							
0	70	151	78	94	70	151	78	94
2	68	165	77.5	83.5	73	153	78	83
4	67	168	77	80	74	154	77	80
6	-	-	-	-	-	-	-	-
8	67	170	76.5	78.5	74.5	154.5	76.5	78.5
10	-	-	-	-	-	-	-	-
12	67	171	76	77.5	75	155	76	77.5

Table 5.20.

ENCLOSURE

Minimum and maximum outside and inside surface temperatures for elements of insulated enclosure, ^{o}F.

Insulation Resistance $= 4^{o}F.hr.ft.^{2}$/Btu	Insulation outside				Insulation inside			
	Outside surface		Inside surface		Outside surface		Inside surface	
	Min	Max	Min	Max	Min	Max	Min	Max
	Indoor Air Temp. Min = 92, Max = 97				Indoor Air Temp. Min = 91, Max = 99			
Roof	66	168	93	98	72	153	91.5	100
West wall	74	140	93.5	96	79	130	92	98
North wall	74	120	92.5	97	76	118	91	99
South wall	74	111	93	96	76.5	107	91.5	98
Glass*	79	104	80	103	79.5	104.5	80.5	104
Floor	-	-	93.5	96	-	-	93	97.5
Partition	-	-	93.5	96	-	-	93	97.5
	Indoor air temperature constant = $75^{o}F$.							
Roof	65	167	78	82	71	152	78	82
West wall	73	139	78	79	77.5	129	78	79
North wall	74	119	77	80	75	117	77	80
South wall	73.5	110.5	78	79	77	105	78	79
Glass*	75	98	75	97	74	98	74.5	97
Floor	-	-	79	80	-	-	79	80
Partition	-	-	77	78.5	-	-	76.5	78

* North and South window glass have the same curve.

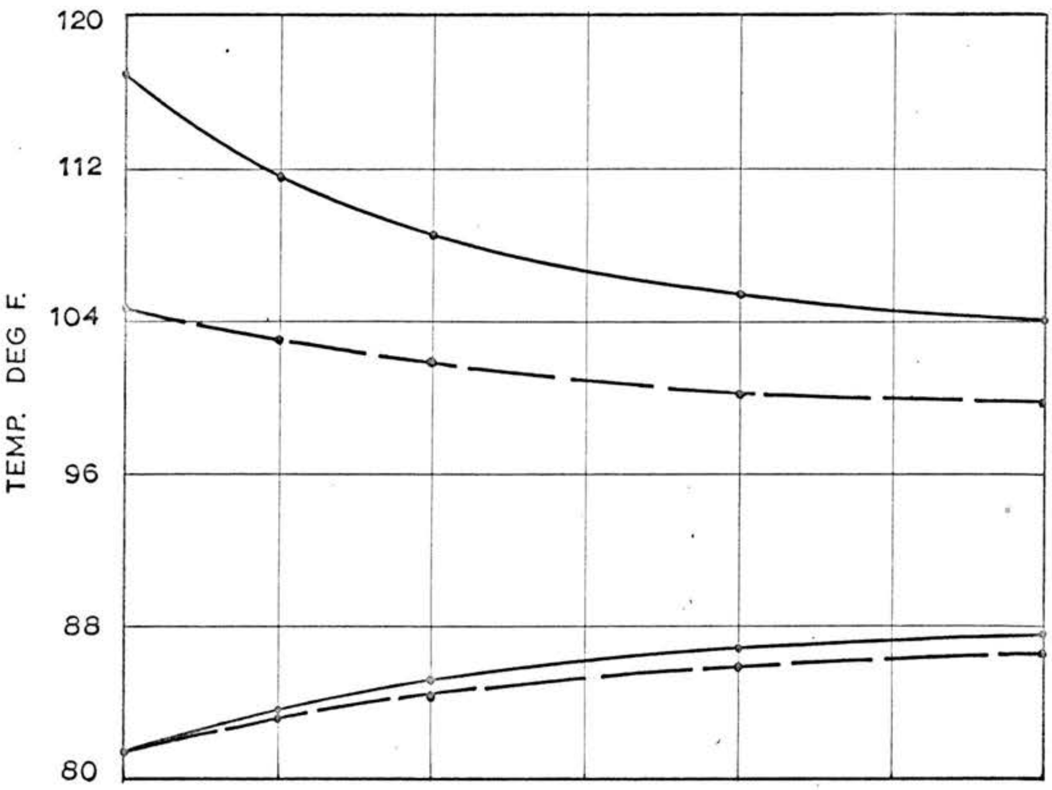
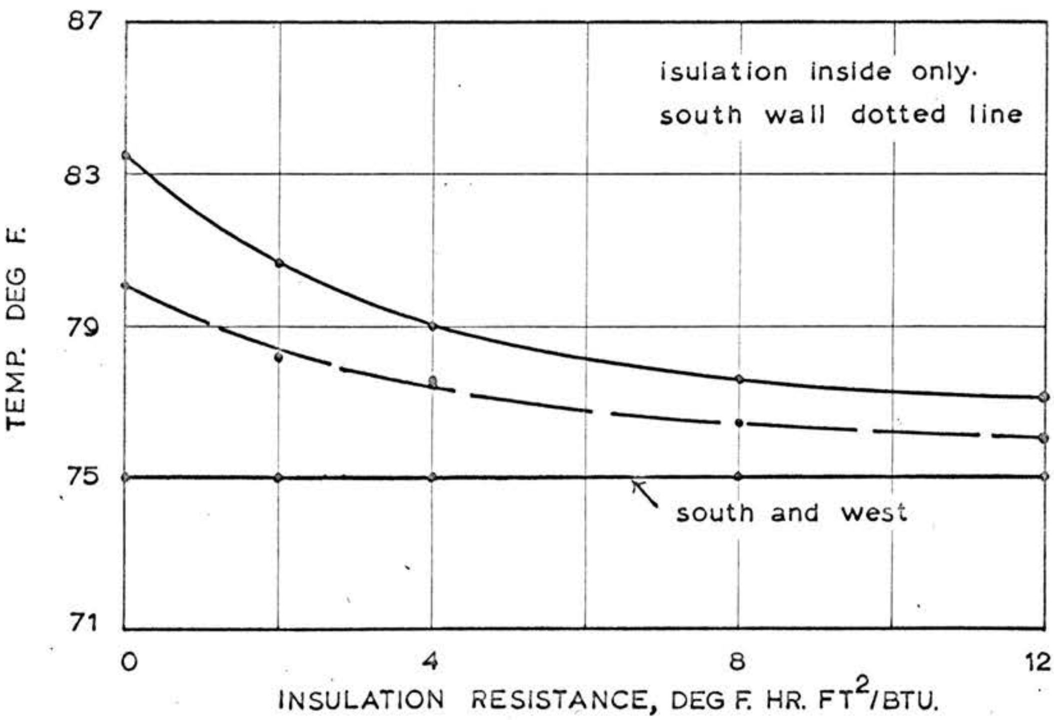


FIG. 43. INDOOR AIR TEMP. VARIABLE.

FIG. 44. INDOOR AIR TEMP. CONSTANT = 75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS.
WITH INSULATION

WEST (AND SOUTH) FACING TIMBER WALL

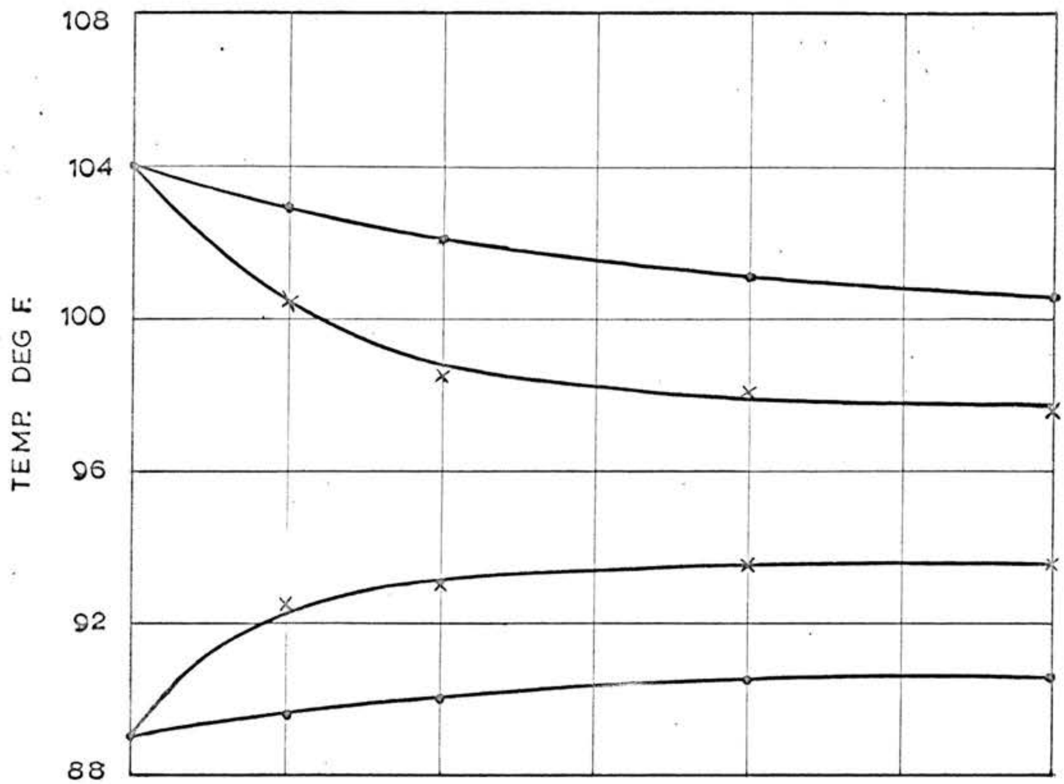
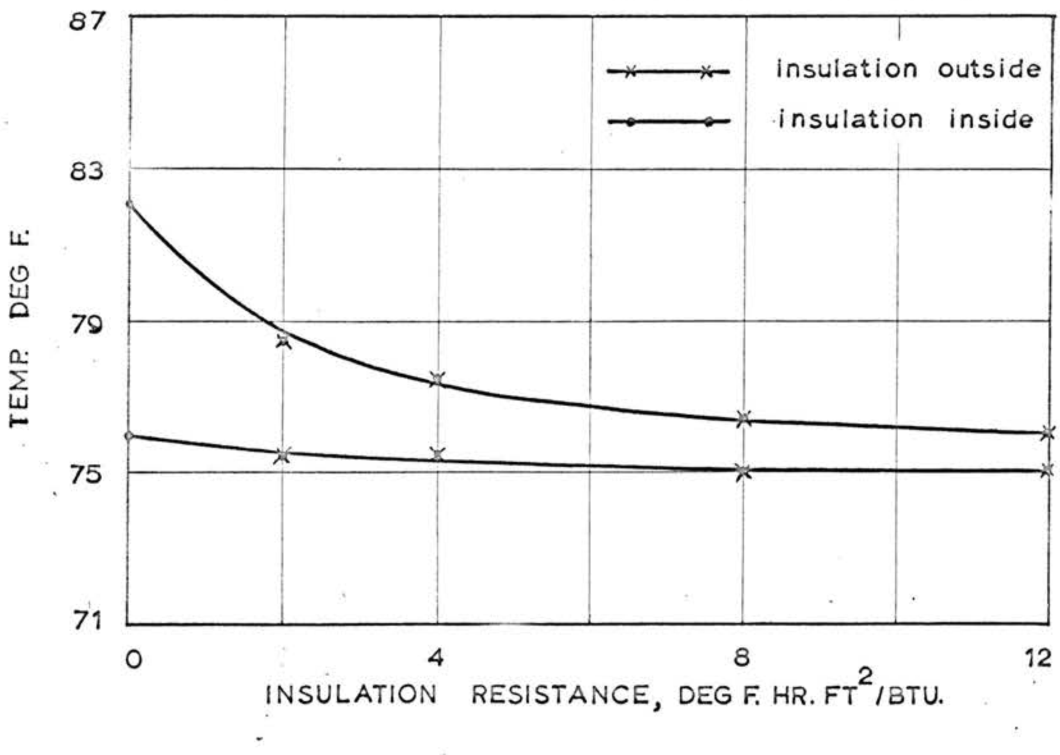


FIG. 45. INDOOR AIR TEMP. VARIABLE

FIG. 46. INDOOR AIR TEMP. CONSTANT = 75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS. WITH INSULATION

WEST FACING HOLLOW BLOCK WALL

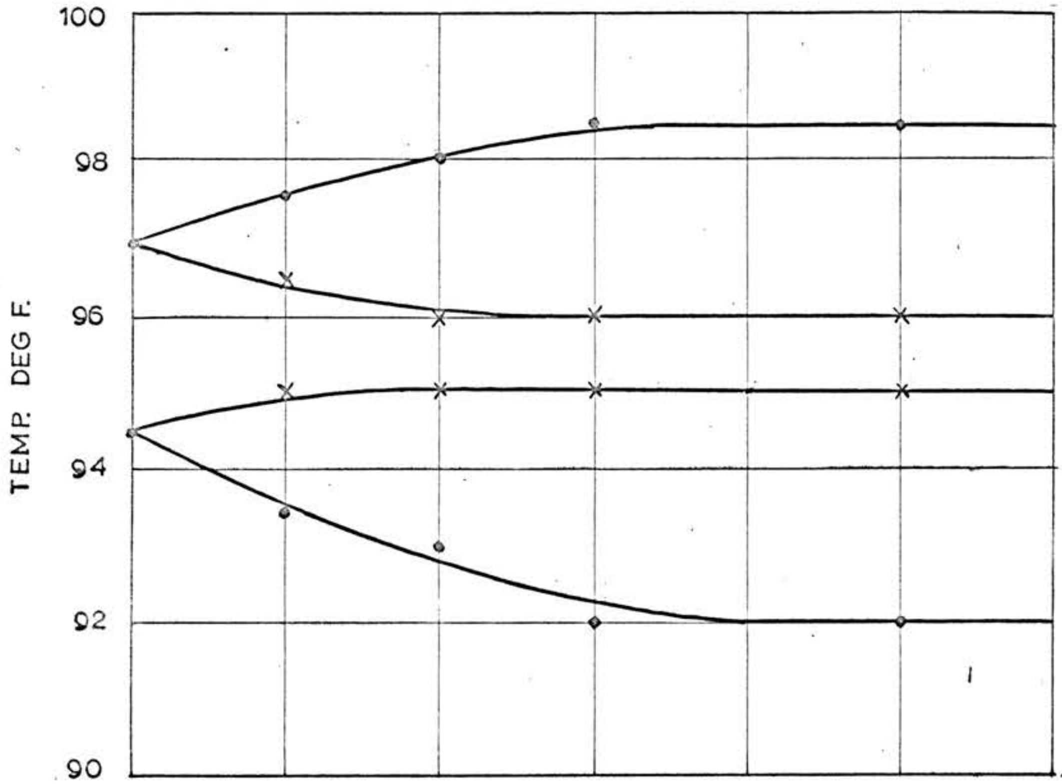
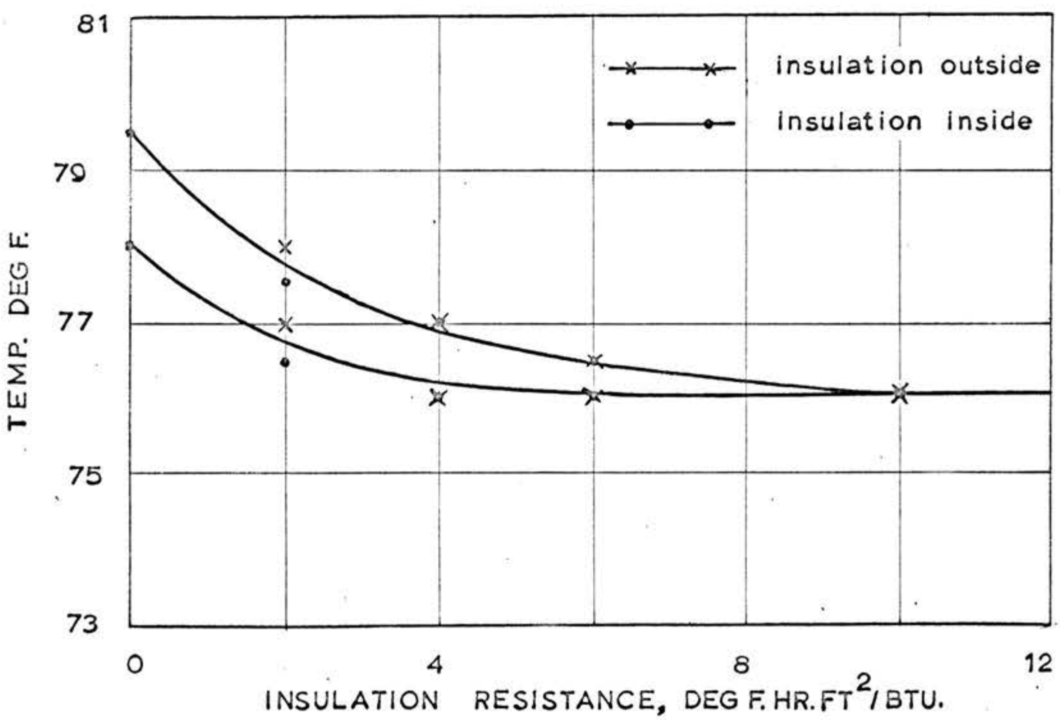


FIG. 47. INDOOR AIR TEMP. VARIABLE.

FIG. 48. INDOOR AIR TEMP. CONSTANT = 75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS.
WITH INSULATION

WEST FACING BRICK WALL

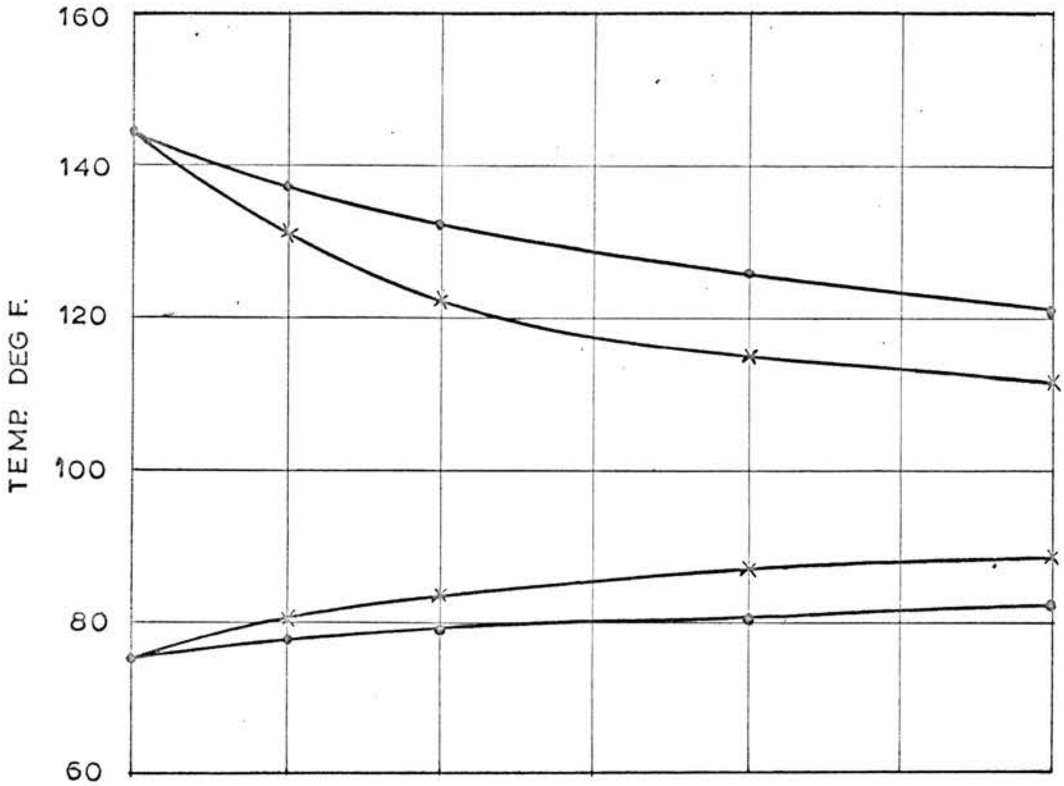
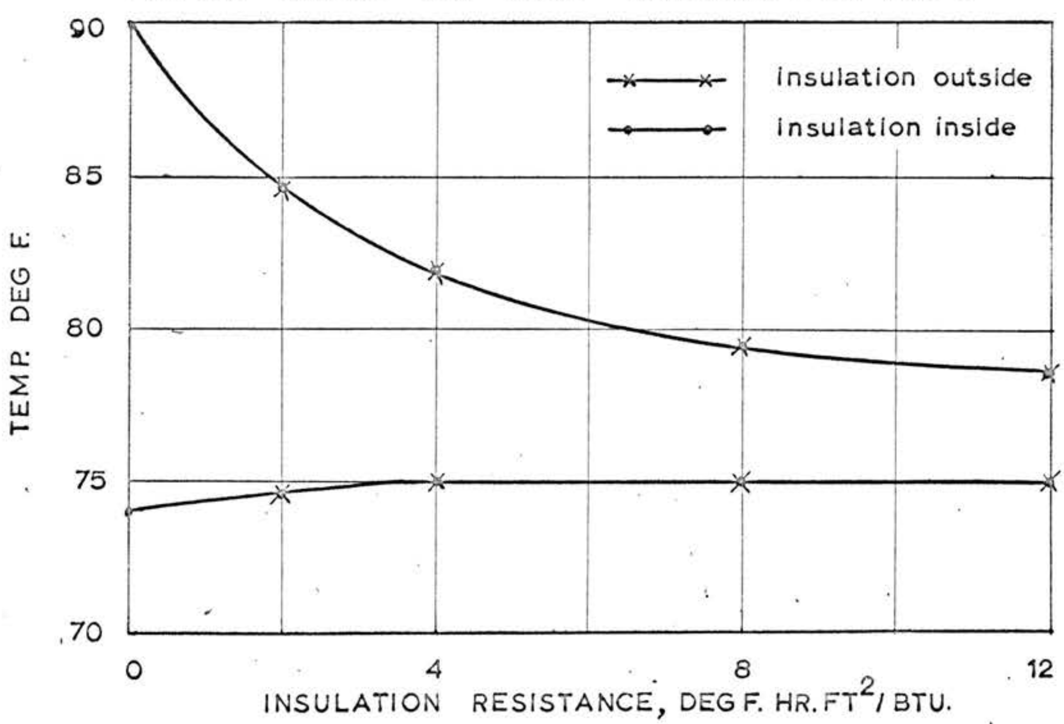


FIG. 49. INDOOR AIR TEMP. VARIABLE

FIG. 50. INDOOR AIR TEMP. CONSTANT = 75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS.
WITH INSULATION

TIMBER ROOF.

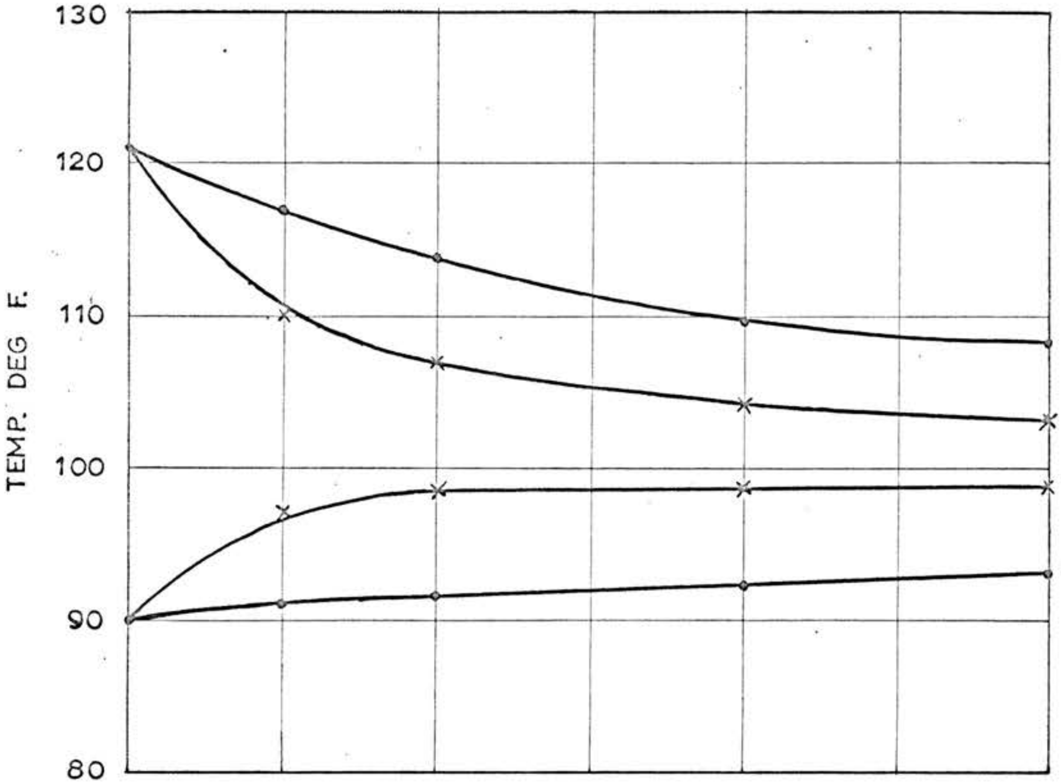
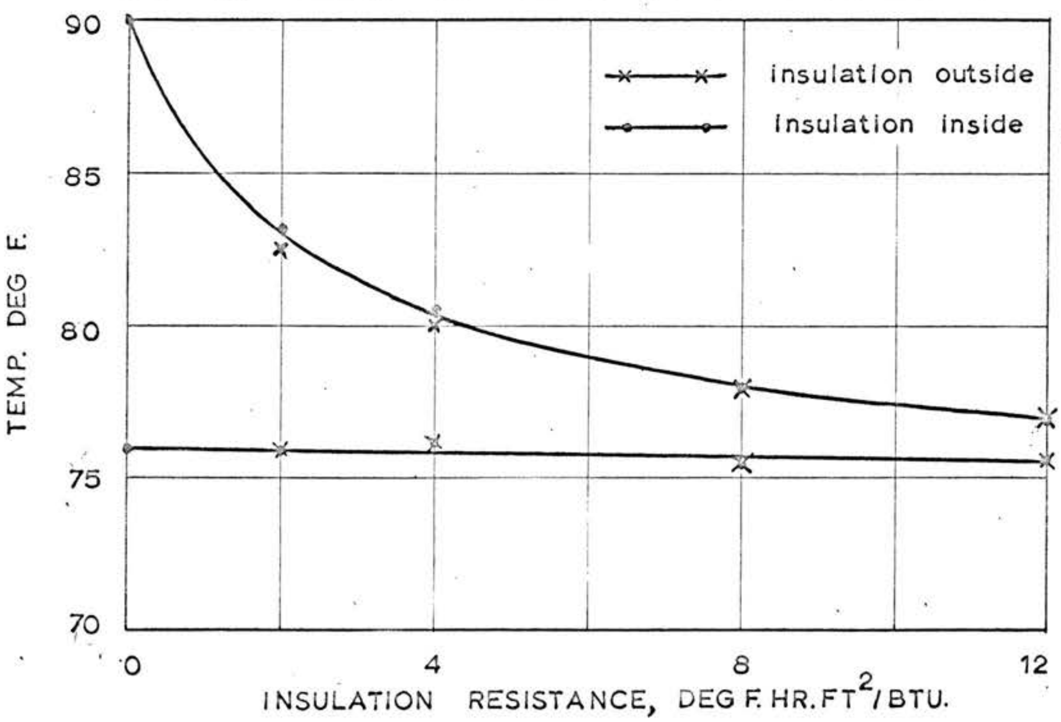


FIG. 51. INDOOR AIR TEMP. VARIABLE

FIG. 52. INDOOR AIR TEMP. CONSTANT = 75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS. WITH INSULATION

HOLLOW TROUGH ROOF

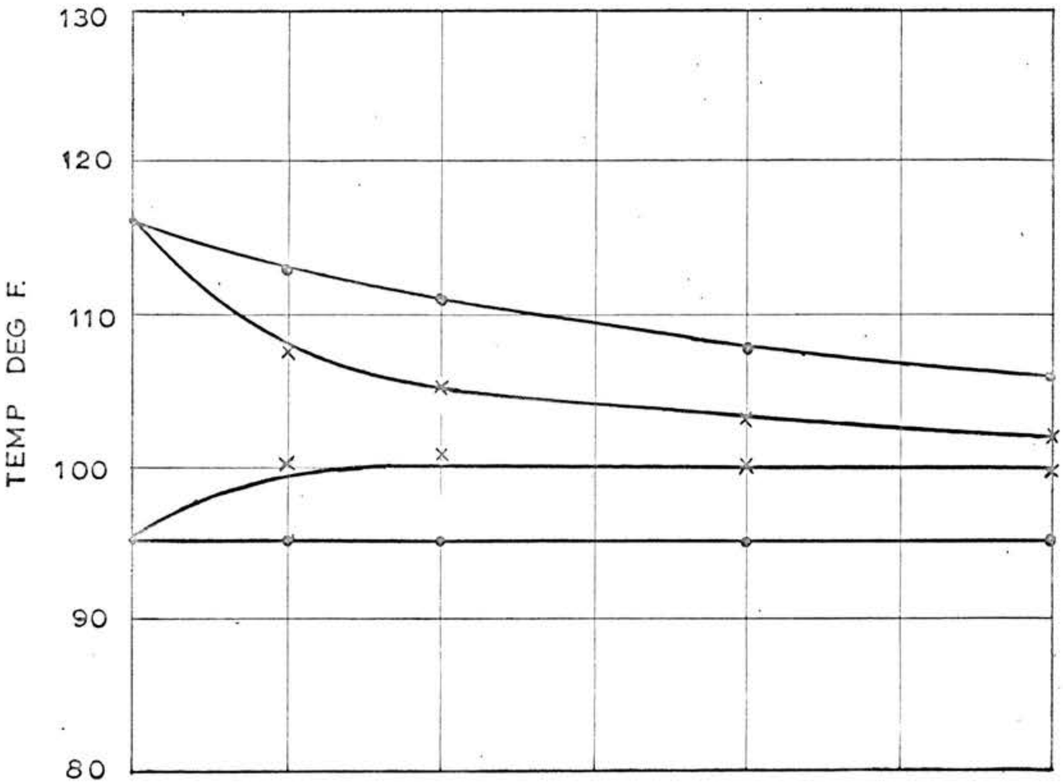
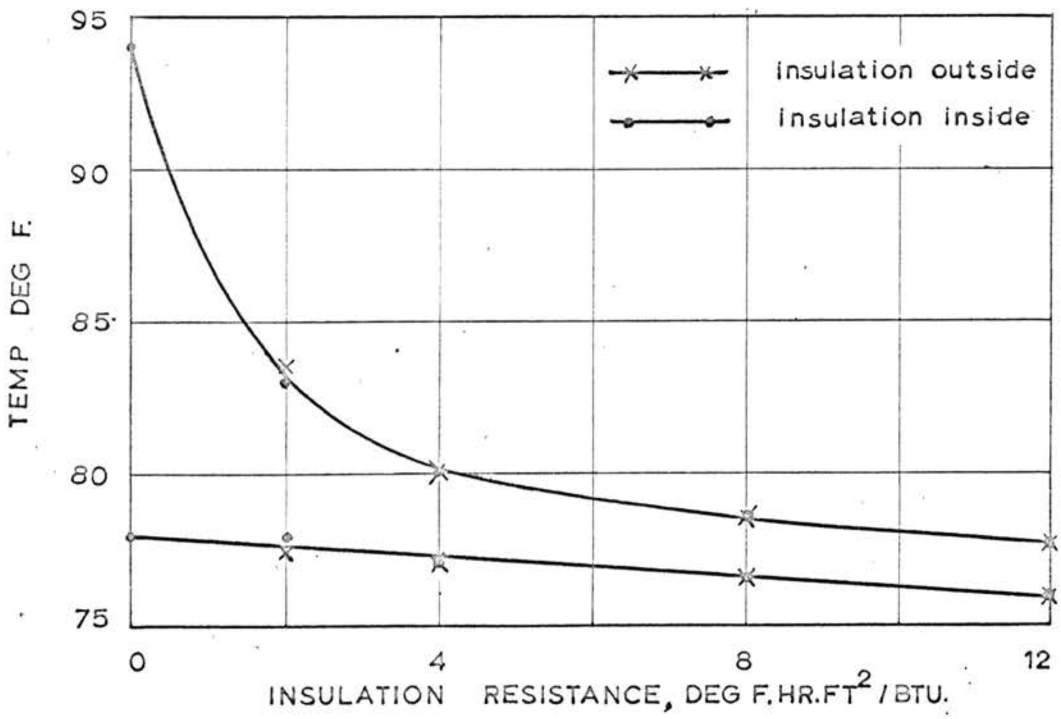


FIG. 53. INDOOR AIR TEMP. VARIABLE

FIG. 54. INDOOR AIR TEMP. CONSTANT =75 DEG F.



VARIATION OF MAX. AND MIN. INSIDE SURFACE TEMPS. WITH INSULATION

SOLID CONCRETE ROOF

5.2.2.2. Inside surface temperatures

When the indoor air temperature is assumed variable, Figs (43, 45, 47, 49, 51 and 53), and Tables 5.5 - 5.19, bring out the effect of placing of the insulation, on the inside surface temperatures. Whereas outside insulation seems to reduce inside surface temperature peaks and temperature ranges, differences are found in the way elements react to inside insulation. For the timber wall no difference was found, as has been mentioned earlier, in surface temperature for outside and inside insulation. For the hollow block wall and roofs, inside insulation seems less effective in reducing peak inside surface temperatures, or restricting the temperature ranges. In the case of the brick wall the effect of inside insulation is adverse, resulting in higher peak temperatures and larger ranges at the inside surface, than for the uninsulated element.

In the enclosure, inside insulation results in higher indoor air and surface temperatures, and larger temperature ranges, than when the insulation is placed outside, as Table 5.20 will show.

When, however, the indoor air temperature is assumed constant at 75 ^{o}F, Figs 44, 46, 48, 50, 52 and 54 and Tables 5.5 - 5.19, the same inside surface temperatures generally obtain for each element whether insulation is placed on the outside or the inside surface. This is confirmed by the temperatures observed for the enclosure, Table 5.20, where surface temperatures of insulated elements remained the same for both positions of the insulation.

Whether outside or inside, however, insulation seems in all cases to result in a reduction in the mean temperatures of inside surfaces, and therefore in the cooling load.

5.2.3. Amount of insulation

When the indoor air temperature is assumed variable, the inside surface temperature range seems to decrease rapidly with the first few increments of insulation, when this is placed on the outside, and more gradually after an insulation resistance of 4 ^{o}F.hr.ft.^{2}/Btu (equivalent to 1 inch of polystyrene) has been applied. Lighter elements, however, retain higher inside surface temperature maxima and larger ranges at the inside surface, for the same amount of insulation, than heavier elements and seem to require more insulation to produce comparable conditions. For example, no reduction in inside surface temperature range was found after an insulation resistance of 4^{o} F.hr.ft.^{2}/Btu is applied to the west facing brick wall, while the west facing hollow block wall showed a reduction in inside surface temperature range for up to an insulation resistance of 8 ^{o}F.hr.ft.^{2}/Btu, and the west facing timber wall, up to 12 ^{o}F.hr.ft.^{2}/Btu.

The change in inside temperature range is less rapid with inside insulation, and when inside insulation does not lead to a larger inside surface temperature range than in the uninsulated element (e.g. brick wall) more insulation is needed to bring about a reduction in inside temperature peak and range than when insulation is placed on the outside.

A comparison is possible between the effects of shading and insulation. For example, while shading the timber wall (compare Table 5.6 for west, and Table 5.8 for south orientations, and see Fig. 43) reduces the inside surface temperature maxima from 117 ^{o}F to 104.5 ^{o}F, it takes the equivalent of 3 inches of polystyrene insulation to reduce the inside surface temperature of the unshaded wall to 104 ^{o}F. Similarly, while shading the hollow block wall (compare Table 5.10 for west, and Table 5.12 for south, orientations) reduces the maximum inside surface temperature from 104 ^{o}F to 98.5 ^{o}F it takes the equivalent of 2 inches of polystyrene, placed on the outside surface, to produce the same reduction in the inside surface temperature peak.

A combination of shading and high capacity, like in the case of the south facing brick wall, Table 5.16, seems to remove the need for insulation, since no change, in inside surface temperatures, seems to occur for (outside) added insulation.

When the indoor air temperature is assumed constant at 75 ^{o}F, Figs 44, 46, 48, 50, 52 and 54, the mean inside surface temperature, in all cases, reduces rapidly with the first few increments of insulation, whether placed on the outside or the inside surface, and more gradually after an insulation resistance of 4 ^{o}F.hr.ft.^{2}/Btu (the equivalent of 1 inch of polystyrene) has been applied.

5.2.4. Discussion

While insulation placed on the outside surface seems to result in a reduction in peak temperatures and temperature ranges at the inside surfaces of building elements, the effect of inside insulation on inside surface temperatures seems to depend on a number of factors.

When insulation is placed on the inside surface of an element acting as a heat sink, this seems to prevent the dissipation of heat due to infiltration and gained from inside heat donor sources. The brick wall and the enclosure illustrate this effect. When insulation is placed inside the higher indoor heat content tends to raise the inside surface temperature peak and increase the temperature range.

On lighter elements, when the heat sink capacity is not so important, and on heat donor sources already with high temperature peaks (e.g. solid concrete roof, Fig. 53.) this adverse effect is less pronounced.

In fact, the reduction in the ceiling temperature peak of the concrete roof, Fig. 53, with inside insulation, seems to be due to a restriction of the ceiling's heat donor property. Hendry[6] noted such an effect on concrete roofs insulated on the underside of the concrete slab. The restriction of heat flow at the underside, he notes, tends to raise the mean temperature of the roof: which seems to explain the lesser effectiveness of inside insulation compared to outside insulation, in the case of donor elements.

Gupta, Gupta, Jain and Raychaudhuri[7] investigating periodic heat flow in conditioned, brick walled, test houses in Roorkee, reported the finding that outside insulation is more effective in reducing the cooling load, than inside insulation. This seems contrary to the observation made earlier, that in these studies outside and inside insulation were found to produce the same inside surface temperatures in conditioned structures. However in the investigation made by the above authors the roofs of the test houses were not insulated. The authors reported another investigation[8]

6. Hendry, O.W., and Page, J.K., "Thermal movements and stresses in concrete slabs in relation to tropical conditions". B.R.S., Institute of Technology, Haifa, July, 1960.
7. Gupta, M.L., Gupta, C.L., Jain, S.P., and Raychaudhuri, B.C., "Periodic heat flow in conditioned structures". Indian Journal of Technology, Vol. 3, October, 1965.
8. Raychaudhuri, B.C., Jain, S.P., Gupta, C.L. and Gupta, M.L., "Heat transmission in insulated masonary structures under unsteady heat flow conditions". Indian Journal of Technology, Vol. 4, March, 1966.

on the same test houses where they found that the uninsulated roof was the controlling element in the heat input to the enclosure, and that the total daily heat input was of the same order whether the walls were insulated or not. Although these observations were made on the test houses when the houses were not conditioned, the contribution of the uninsulated roofs to the finding that outside insulation was more effective than inside insulation, is obvious.

This brings out the importance of planning insulation so that when insulation is applied to the inside surfaces of elements it is applied to heat donors rather than heat sinks ~~sources~~.

In unconditioned buildings, if the purpose of insulation is the provision of comfort conditions, the application and the positioning of insulation should be related to conditions of occupation. Lowering indoor temperature maxima by the use of insulation is usually accompanied by a rise in temperature minima. The times of occurrence of these temperatures are of importance. If, for example, a timber frame enclosure is used only at night the higher minimum night temperatures resulting from insulation may be disadvantageous.

CHAPTER 6

CONCLUSIONS.

The following conclusions can be drawn:

1. In hot dry climates, comfortable thermal conditions cannot be maintained in buildings, under severe summer conditions, without mechanical cooling.
2. Heavy structures effect a greater modification of the outdoor environment, than light structures. Heavy shaded structures seem to give the best thermal performance.
3. Roofs and west facing walls are important elements to be considered in the thermal design of buildings.
4. Insulation placed on the outside surface of a building element, under the climatic conditions considered, and assuming periodic heat flow, produces a reduction in peak inside temperature and lessens the temperature ranges at the inside surface of the insulated element.
5. In unconditioned buildings, insulation, placed on the inside surface of a building element, is less effective in reducing inside peak temperatures, than outside insulation, and may lead to higher peak inside temperatures, and larger temperature fluctuations, depending on the heat sink capacity of the structure and the magnitude of internal heat gain.

6. In conditioned buildings, where the indoor air temperature is kept constant, insulation, whether placed on the outside or inside surfaces of building elements, has the same effect on, and tends to reduce the mean of, the inside surface temperatures.

7. Insulation equivalent to one inch of expanded polystyrene, appropriately ~~approximately~~ placed, seems adequate for heavy structures. The lighter the structure, the more the insulation needed to produce comparable indoor conditions.

8. When designing thermal insulation attention should be given to the conditions of occupation.

APPENDIX 1

Comparison of Analytical and Analogue Solutions of Periodic Heat Flow in a Wall

The case considered is that of a wall perfectly insulated on one side, and subjected on the other to a periodic sinusodial temperature variation.

The interest lies in the attenuation of the temperature wave as it traverses the medium, expressed in the amplitude ratio of the temperature variations at the two surfaces, and the time lag between the occurrence of the temperature maxima on the two surfaces.

The analytical solution* is compared with the analogue solution of the same problem. The purpose of the experiment is to determine whether the analogue can give a reliable prediction.

(1) Analytical Solution

The Fourier conduction equation for one dimensional, unsteady, heat flow is

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

It is convenient to place the wall in the region o x 1, with the insulated surface at x = o, and the surface at x = 1 subjected to a sinusodial temperature variation. This is done to make the algebra simpler.

To solve equation (1) we attempt a solution of the form:

$$T_{x,t} = Ae^{(a + ib)x}e^{iwt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

---

* Analytical solution provided by Dr. M.G. Davies of the Department of Building Science, the University of Liverpool.

and we take the real part of equation (2) to represent the solution.

This takes account of the following features:

(1) At any position x, in the wall, the temperature varies sinusodially - hence the (e^{iwt}) term.

(2) For the spatial part of the solution, the waves are attenuated as they traverse the medium - this is accounted for by the term (e^{ax}), and because of the choice of the direction of x, (a) is positive.

However, when a wave reaches the insulated surface it is reflected back on itself. Since there are phase relationships involved, the reflected wave will interfere with the incident part of the wave, thereby setting up a region of standing waves. Hence the need for the term (e^{ibx}). Eventually, when x is large enough, the reflected wave will become so attenuated in comparison with the incident wave that it can be neglected. Beyond this point the temperature depends only on the incident wave, and the response is identical with that for a semi-infinite medium.

(3) (A) will have units of temperature.

ax, bx and wt are dimensionless.

<u>Solution</u>:

Differentiate equation (2) with respect to x twice and t once and substitute in equation (1). Then

$$(a + ib)^2 = \frac{iw\, c \rho}{k}.$$

We equate the real and imaginary parts and find

$$a^2 - b^2 = 0;$$

$$2ab = \frac{w\, c \rho}{k}.$$

Case I: $a = +b$ and $a = \pm \sqrt{\dfrac{w\,c\,\rho}{2k}}$

Case II: $a = -b$ and $a = \pm \sqrt{\dfrac{-w\,c\,\rho}{2k}}$

But w is inherently positive, also ρ, c, and k, so we reject case II

Write $\sqrt{\dfrac{w\,c\,\rho}{2k}} = +m$ .... (3)

Hence case I yields two solutions ($a = +m$, and $a = -m$):

(1) $T_{x,t} = A_1 e^{mx} e^{imx} e^{iwt}$

(2) $T_{x,t} = A_2 e^{-mx} e^{-imx} e^{iwt}$

Since equation (1) is linear in T, we can superimpose any basic solutions to produce a composite solution. So we write

$$T_{x,t} = A_1 e^{mx} e^{imx} e^{iwt} + A_2 e^{-mx} e^{-imx} e^{iwt} \quad \text{............... (4)}$$

Now we introduce the boundary conditions. These are that we know the actual temperature, at any time, at $x = o$, and that at $x = o$, the heat flux is zero. (In practice complete knowledge of T at $x = 1$, would be assumed and T found at $x = o$, but it is more convenient here to assume T at $x = o$, and find it at $x = 1$.)

At $x = o$, $T = T_o e^{iwt}$; $\therefore A_1 + A_2 = T_o$ (from equation 4)

At $x = o$, $\dfrac{\partial T}{\partial x} = o$, at all times.

Now $\dfrac{\partial T}{\partial x} = A_1(me^{mx} e^{imx} e^{iwt} + ime^{mx} e^{imx} e^{iwt})$

$+ A_2(-me^{mx} e^{imx} e^{iwt} - ime^{-mx} e^{-imx} e^{iwt}) = 0$

$\therefore A_1 - A_2 = o$

$\therefore A_1 = A_2 = \tfrac{1}{2} T_o$

The solution is

$$\frac{T_{x,t}}{T_o} = \tfrac{1}{2}(\underbrace{e^{mx}e^{i(wt + mx)}}_{\text{part one}} + \underbrace{e^{-mx}e^{i(wt - mx)}}_{\text{part two}}) \quad \ldots\ldots\ldots\ldots (5)$$

Part one of the solution represents a wave travelling from the outside surface to the inside surface ($e^{i(wt + mx)}$) lying within the envelope $\pm\, e^{mx}$. Part two represents a wave travelling in the opposite direction ($e^{i(wt - mx)}$) lying within the envelope $\pm\, e^{-mx}$.

It is more convenient to express equation (5) in trigonometrical and hyperbolic functions.

$\frac{1}{m}$ or $\sqrt{\frac{Pk}{\pi c}}$ (P = periodic time, i.e. 24 hours) defines a "characteristic length" for the problem; the distance in which, in the semi-infinite wall, the amplitude falls to $\frac{1}{e}$ or 0.37 of its initial value.

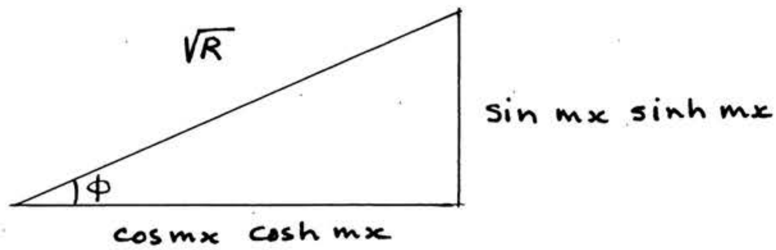
We express the real part:

$$\frac{2T_{x,t}}{T_o} = e^{mx}(\cos wt + i \sin wt)(\cos mx + i \sin mx)$$
$$+ e^{-mx}(\cos wt + i \sin wt)(\cos mx - i \sin mx).$$

$$\frac{T_{x,t}}{T_o} = \cos wt.\ \cos mx \frac{(e^{mx} + e^{-mx})}{2}$$
$$+ \sin wt.\ \sin mx \frac{(-e^{mx} + e^{-mx})}{2}$$
$$+ \text{imaginary part.}$$

$$\frac{T_{x,t}}{T_o} = \cos wt.\ \cos mx.\ \cosh mx + \sin wt.\ \sin mx.\ \sinh mx$$

Transforming by multiplying and dividing by the sum of squares of terms in mx



$$R = \cos^2 mx \frac{(e^{2mx} + e^{-2mx} + 2)}{4} + \sin^2 mx \frac{(e^{2mx} + e^{-2mx} - 2)}{4}$$

$$= \frac{1}{4}(e^{2mx} + e^{-2mx} + 2 \cos 2 mx)$$

Write

$$\cos \phi = \cos mx . \cosh mx / \sqrt{R}$$

$$\sin \phi = \sin mx . \sinh mx / \sqrt{R}$$

then,

$$\frac{T_{x,t}}{T_o} = \frac{1}{2}\sqrt{e^{2mx} + e^{-2mx} + 2 \cos 2mx} \; . \cos (wt-\phi) \;..... (6)$$

and, $\tan \phi = \tan mx . \tanh mx$ .... (7)

This is the final solution.

<u>Interpretation</u>:

The solution represents an incident wave e^{2mx}, a reflected wave e^{-2mx}, and their interference 2 cos2mx, to produce standing waves.

The phase lag between an imposed temperature maximum at x = 1 and its arrival at x = o is given by $\cos (wt - \phi)$.

$T_{x,t}$ is maximum when:

$$\cos (wt - \phi) = 1$$

or, $\phi = wt$

assuming we consider the same wave crest.

The time lag, $t = \frac{\Phi}{w}$ .... (8)

Application:

The test wall has the following specification:

Density, $\rho = 144$ lb./ft^3.

Specific heat, $c = 0.21$ Btu/lb.°F.

Conductivity, $k = 0.42$ Btu.ft./ft^2.hr.°F.

Area $A = 200$ sq. ft.

Thickness, $l = 0.75$ ft.

Choosing a time when $T_{x,t}$ is maximum, i.e. when $\cos(wt - \Phi) = 1$,

equation (6) gives the amplitude ratio as

$$\frac{T_{x,t}}{T_o} = \tfrac{1}{2}\sqrt{e^{2mx} + e^{-2mx} + 2\cos 2mx} \text{ .... (9)}$$

Now, $m = \sqrt{\frac{w c \rho}{2k}}$.... equation (3)

and, $w = \frac{2\pi}{P}$ (where P, period = 24 hrs.)

$w = \frac{\pi}{12}$

Substituting in equation (3)

$$m = \sqrt{\frac{\pi \times 144 \times 0.21}{12 \times 2 \times 0.42}} = 3.07$$

$2mx = 2 \times 3.07 \times 0.75 = 4.605$ radians

$e^{2mx} = e^{4.6} = 99.48$

and, $e^{-2mx} = e^{-4.6} = 0.01$

and, $\cos 2mx = \cos 4.605 = -0.107$

$2\cos 2mx = -0.214$

Writing $T_{x,t}$ as T_1 for the outside surface, and substituting in equation (9), we have:

$$\frac{T_1}{T_o} = \frac{1}{2}\sqrt{99.48 + 0.01 - 0.214} = 4.982$$

Taking the reciprocal, we have the ratio of the temperature amplitude on the insulated side to that on the outside surface as

$$\frac{T_o}{T_1} = \frac{1}{4.982} = \underline{\underline{0.201}}$$

The time lag is given by equation (8) as

$$t = \frac{\phi}{w} \quad (\text{where, } w = \frac{\pi}{12})$$

and ϕ, the phase angle, is given by equation (7) as

$$\tan\phi = \tan mx . \tanh mx.$$

Now, $mx = 2.303$ radians

and, $\tan mx = \tan 2.303 = -1.113$

and, $\tanh mx = \tanh 2.303 = 0.98$

$\therefore \tan\phi = -1.113 \times 0.98 = -1.09$

or $\phi = \pi - 0.829 = 2.31$ radians

time lag, $t = \dfrac{2.31 \times 12}{3.142} = \underline{\underline{8.8 \text{ hours}}}$

(2) <u>Analogue Solution</u>

The circuit parameters are determined as follows:

Thermal resistance of the wall (R_t) is given by

$$R_t = \frac{1}{kA}$$

where, $l = 0.75$ ft.

$k = 0.42$ Btu ft./ft^2.hr.$^\circ$F.

$A = 200$ sq. ft.

$$R_t = \frac{0.75}{0.42 \times 200} \sim 0.009 \text{ deg. F. hr/Btu.}$$

The electrical resistance (R_e) is given by

$$R_e = R_t \, N_R \text{ ohms}$$

and N_R, the scale factor $= 1 \times 10^8$

$$R_e = 0.009 \times 10^8 \text{ ohms} = \underline{900 \text{ K.ohms}}$$

The thermal capacity (C_t) is given by

$$C_t = \rho c V$$

where, $\rho = 144$ lb./ft^3.

$c = 0.21$ Btu/lb.$^\circ$F.

$V = 200 \times 0.75$ cu. ft.

$$C_t = 144 \times 0.21 \times 200 \times 0.75 = 4536 \text{ Btu}/^\circ\text{F}$$

The electrical capacitance (C_e) is given by

$$C_e = C_t . N_C \text{ farads}$$

and N_C, the scale factor $= 1 \times 10^{-8}$

$$C_e = 4536 \times 10^{-8} \quad \underline{45 \text{ microfarads}}$$

The network is made up of 45 T-sections, each with a resistance of 20 K.ohms, and a capacitance of 1 microfarad.

The boundary conditions imposed are as follows:

At the node representing the outside surface ($x = 1$), a voltage varying sinusoidally between zero and 10 volts is applied through a resistance of 125 K.ohm representing an outside surface film conductance of 4.0 Btu/ft^{2}hr.$^\circ$F.

Since the wall is assumed to be perfectly insulated at the inside surface ($x = o$), it follows that there is no heat flow through the surface. This condition is satisfied by leaving the circuit open at the node representing the inside surface.

Solution:

The voltage variation on both surfaces is measured and the amplitude ratio determined.

The voltage on the outside surface ($x = 1$), is found to vary between 1.4 and 8.6 volts. On the inside surface ($x = o$), the range is 4.3 to 5.7 volts (see trace).

The amplitude ratio is, therefore:

$$\frac{5.7 - 4.3}{8.6 - 1.4} = \frac{1.4}{7.2} = \underline{\underline{0.194}}$$

The time lag is found by comparing the distance on the chart, between the occurrence of maximum voltage on the outside and inside surfaces, to the length of a 24 hour trace, (see trace).

The distance between the two maxima is $1\frac{1}{4}$ inches. A 24 hour trace is $3\frac{1}{3}$ inches.

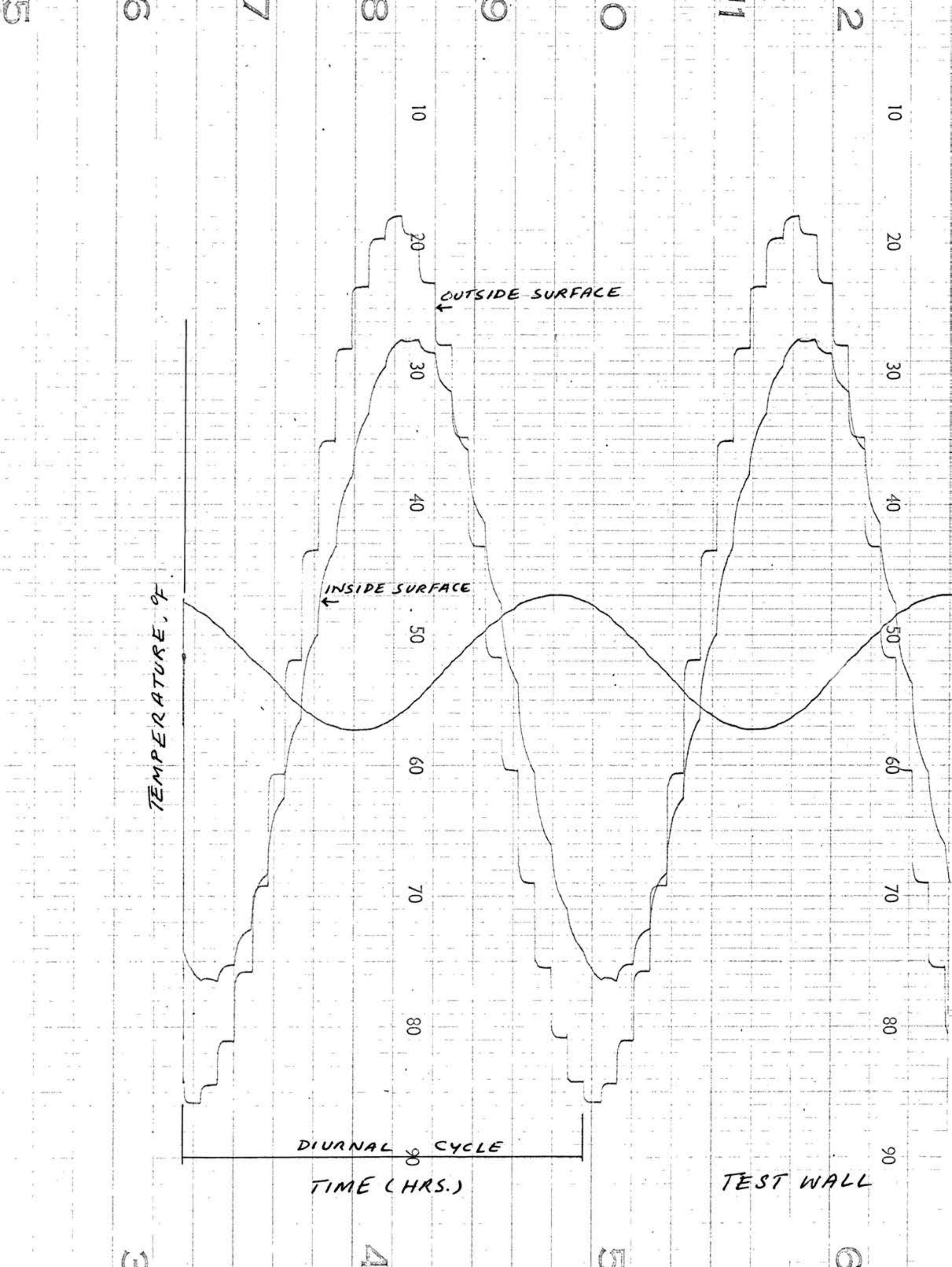
The time lag is, therefore:

$$\frac{5}{4} \times \frac{3}{10} \times 24 = \underline{\underline{9.0 \text{ hours}}}$$

Comparison:

The analogue prediction of amplitude ratio (0.194) is at $3\frac{1}{2}\%$ variance with the analytical solution (0.201). The analogue time lag (9.0 hours) is 0.2 hours more than the analytical prediction (8.8 hours), a difference of about $2\frac{1}{2}\%$.

This is considered to be reasonable agreement. Although this does not, necessarily, mean that all other analogue predictions are within this degree of accuracy, it is taken as an indication that meaningful results can be obtained using the analogue.



OUTSIDE SURFACE
INSIDE SURFACE
TEMPERATURE, °F
DIURNAL CYCLE
TIME (HRS.)
TEST WALL
10
20
30
40
50
60
70
80
90

APPENDIX 2

Estimated Summer Ground Temperatures

for Khartoum

Exposed ground receives solar radiation, and is in heat exchange by long wave radiation with the sky, and convection with the outdoor air. Saad and Hendry*, who measured soil temperatures at different depths, in Khartoum, reported that diurnal temperature variations diminish rapidly with depth and become insignificant below 9 inches. At a depth of two to three feet they indicated that the temperature can be taken as constant, and in the warm month of May, about 95°F.

As such, the ground can be treated as a semi-infinite slab subjected to the outdoor climate at the upper surface and remaining at constant temperature at the lower side

Circuit parameters

Physical properties of soil, as given in Table 4.7, are :

thermal conductivity, k	= 6.0 Btu.in./ft².hr.°F.
density,	= 100 lb/ft³.
specific heat, c	= 0.2 Btu/lb.°F.
solar absorptivity,	= 0.7
long wave emissivity,	= 0.9

The depth of soil considered is l = 2.5 feet. An area of 200 sq. ft is assumed for the calculations.

The total thermal resistance is given by equation (2.18) as:

$$R_t = \frac{1}{kA} = \frac{2.5 \times 12}{6.0 \times 200} = 0.025 \ ^\circ F.hr./Btu.$$

---

* loc.cit., see footnote 4.9.

The electrical resistance is given by equation (2.19) as:

$$R_e = R_t.N_R = 0.025 \times 10^8 = \underline{2500} \text{ Kilo-ohm.}$$

The total thermal capacity is given by equation (2.22) as:

$$C_t = \rho clA = 100 \times 0.2 \times 2.5 \times 200 = 10{,}000 \text{ Btu/}^{\circ}\text{F.}$$

The electrical capacitance is given by equation (2.23) as:

$$C_e = C_t.N_C = 10{,}000 \times 10^{-8} = \underline{100} \text{ microfarads}$$

The simulator network is, therefore, made of 25 T-sections, each having a resistance of 100 Kilo-ohms and a capacitance of 4 microfarads.

Boundary conditions

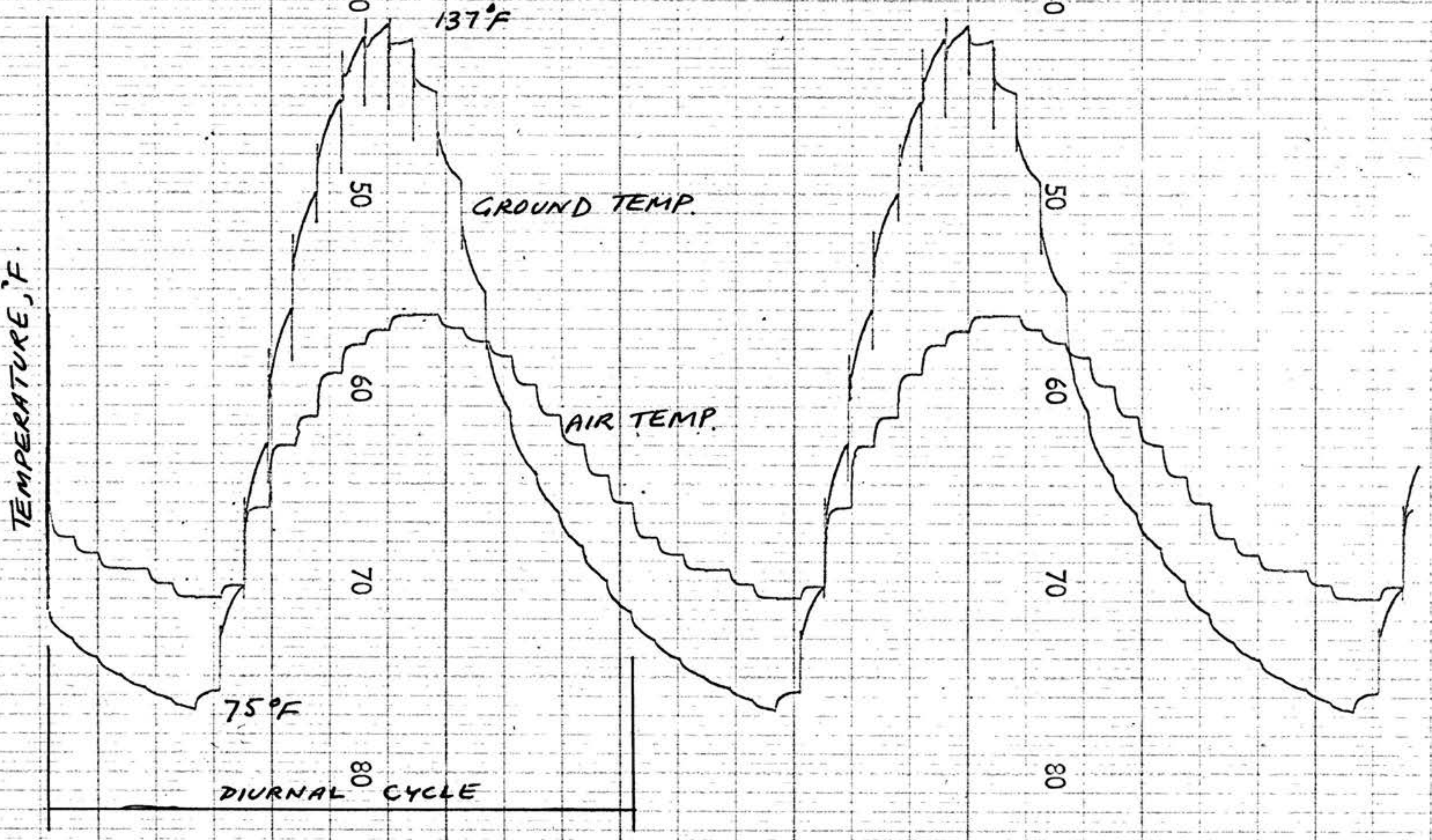
At the upper surface the ground has the same solar radiation input as that shown for a horizontal surface in Table 3.6.. Long wave radiation exchange is assumed to take place with the sky at the temperatures given in Table 3.7.. The outdoor air temperatures, for convective heat exchange, are given in Table 3.2..

The surface coefficient for convective heat exchange, h_{co}, is taken as 2.0 Btu/ft^2.hr.$^{\circ}$F. and the coefficient for radiative heat exchange, h_{ro}, is taken at 1.1 Btu/ft^2.hr.$^{\circ}$F, as discussed in sections 4.4.1 and 4.4.2.

The under side is connected to a constant potential representing 95°F.

Solution

The analogue solution for the hourly temperatures of the upper surface is given in Table 3.8. The analogue output plot is attached.



GROUND TEMPERATURE.

APPENDIX 3

The Variation of Inside Surface Temperatures, and Indoor Air Temperatures, with Ventilation rate.

An expression was given, in section 4.2.3.2, for the heat flux due to air leakage or ventilation, in equation (4.5). The ventilation, or leakage, resistance (R_v) derived from that expression, was given, assuming a room volume of 2000 cu. ft.*, by equation (4.7) as

$$R_v = \frac{1}{40n} . 10^8, \text{ ohms.}$$

where, n = number of air changes per hour.

The indoor air temperature and the outdoor air temperature potentials are connected across the resistance (R_v), as shown in Fig. 8 (Chapter 4) for single elements, and Fig. 15 (Chapter 4) for the enclosure.

Values of the resistance (R_v) for a number of selected rates of air change are given below:

n	R_v, Kilo-ohms
	00
10	250
2.5	1,000
1	2,500
0.25	10,000
0.1	25,000

Investigations:

The variation of indoor air temperatures and inside surface temperatures, was observed for single elements and the enclosure, as the rate of ventilation is varied.

---

* Actually the round figures in equation (4.7) represent a room volume slightly larger than 2000 cu.ft. and slightly smaller than the volume of the enclosure (2350 cu.ft.)

The following trends were found:

1 - When an infinite number of air changes is assumed, i.e. the indoor air, and outdoor air temperatures assumed equal, a reduction occurs in peak surface temperatures of elements which, at lower rates of ventilation, attain higher surface temperature maxima than the outdoor air (111 $^{\circ}$F). For example, the temperature range, at the inside surface, changes for the west facing timber wall, from 81.5 - 117 $^{\circ}$F, at one air change per hour, to 81.5 - 114 $^{\circ}$F at $n = \infty$. The change for the timber roof is from 75 - 144 $^{\circ}$F to 80 - 119 $^{\circ}$F, for the hollow trough roof from 91 - 121 $^{\circ}$F to 84 - 115 $^{\circ}$F, and for the solid concrete roof from 95 - 116 $^{\circ}$F to 88 - 113 $^{\circ}$F. Elements which, at a ventilation rate of one air change per hour, maintain inside surface temperature maxima lower than that of the outdoor air, suffer a rise in inside surface peak temperatures. Thus the temperature range at the inside surface of the west facing hollow block wall changes from 89 - 104 $^{\circ}$F, at $n = 1$, to 84 - 109 $^{\circ}$F at $n = \infty$, and the range at the inside surface of the west facing brick wall changes from 94.5 - 97 $^{\circ}$F, at $n = 1$, to 88 - 103 $^{\circ}$F, at $n = \infty$.

2 - In the enclosure*, the indoor air temperature range diminished as the rate of ventilation was reduced until a value of $n = 1$ is reached. At ventilation rates below one air change per hour no change occurred in the indoor air temperature.

The observed variation of the indoor air temperature with ventilation rate was as follows:

---

* The enclosure is described in section 4.3.1.1.

n	indoor air temperature range, °F.
10	88 - 105
2.5	90 - 103
1	92 - 102
0.25	92 - 102
0.1	92 - 102

The inside surface temperatures of enclosure elements behaved in a similar manner, increasing in range with ventilation rates $n > 1$, and not affected by any reduction in ventilation rate below $n = 1$.

At one air change per hour, the surface temperatures of enclosure elements were given in Table 5.3 (Chapter 5). The surface temperature ranges are given below, for purposes of comparison, at 10 air changes per hour.

Enclosure element	Inside surface temperature range, °F.
Ceiling	89 - 109
West wall	92 - 100
North wall	89 - 104
South wall	91 - 101
Floor	92 - 101
Partition	91.5 - 101.5
Window glass	79.5 - 105.5

3 - A similar effect was observed in the case of single elements tested. The decrease of ventilation rate below one air change per hour seemed to have little effect on inside temperatures.

The solid concrete roof and the west facing brick wall, demonstrate this tendency.

The variation of the inside surface temperature range, for the concrete roof, with ventilation rate is given below:

n	Inside surface temperature range, $^{\circ}F$
	88 - 113
2.5	93 - 115
1	95 - 116
0.25	95.5 - 116.5
0.1	96 - 117

For the west facing brick wall the variation of the inside surface temperature range with ventilation rate was as follows:

n	Inside surface temperature range, $^{\circ}F$
	88 - 103
2.5	92 - 99
1	94.5 - 97
0.25	94.5 - 97
0.1	94.5 - 97

To restrict the number of variables, the insulation studies were made for a fixed ventilation rate. From the above results it seemed that a rate of one air change per hour was a reasonable value to adopt. At higher rates of ventilation indoor air and surface temperatures would follow the outdoor air temperature to a degree which would make the effect of insulation difficult to assess. Also, it is unlikely that the rate of 'natural' air infiltration in an ordinary building would be less than one air change per hour.

The analoque, however, had no facility for varying the ventilation resistance (R_v) during a run, and it was not, therefore, possible to investigate the effect of regulated (timed) ventilation.

List of References.

CHAPTER 1.

1.1. Billington, N.S., "Thermal properties of buildings". Cleaver-Hume Press Ltd., 1952.

1.2. Jaegar, J.C., "An introduction to applied mathematics". Oxford University Press, 1963.

1.3. Mackey, C.O. and Wright, L.T.Jr., "Summer comfort factors as influenced by the thermal properties of building materials". A.S.H.V.E. Transactions, Vol. 49, 1943.

1.4. Roux, A.J.A. "Periodic heat flow through building components - Heat exchange at the outside surface with special reference to the application of sol-air temperature". N.B.R.I. Pretoria, S. Africa. 1950.

1.5. Page, J.K., "A technical note on the calculation of temperatures in concrete roof slabs". RILEM symposium on concrete and reinforced concrete in hot countries, Haifa, 1960.

1.6. Houghten, F.C. and Zobel, C.G., "Heat transfer through a roof under summer conditions". A.S.H.V.E. Transactions, Vol. 34, 1928.

1.7. Houghten, F.C., and Zobel, C.G., "Coefficients of heat transfer as measured under natural weather conditions". A.S.H.V.E. Transactions, Vol. 34, 1928.

1.8. Houghten, F.C., and Zobel, C.G., et al., "Additional Coefficients of heat transfer under natural weather conditions". A.S.H.V.E. Transactions, Vol. 35, 1929.

1.9. Alford, J.S., Ryan, J.E., and Urban, F.O., "Effect of heat storage and variation in outdoor temperature and solar intensity on heat transfer through walls". A.S.H.V.E. Transactions, Vol. 45, 1939.

1.10. Houghten, F.C., Blackshaw, J.L., Pugh, E.M. and McDermott, P., "Heat transmission as influenced by heat capacity and solar radiation". A.S.H.V.E. Transactions, Vol. 38, 1932.

1.11. Mackey, C.O., and Wright, L.T.Jr., "Periodic heat flow - Homogeneous walls or roofs". A.S.H.V.E. Transactions, Vol. 50, 1944.

1.12. Roux, A.J.A., "A test room for the study of heat transmission through building components under conditions of periodic heat flow". N.B.R.I., Pretoria, S. Africa, 1949.

1.13. Roux, A.J.A., "Periodic heat flow through building components - Heat exchange at the inside surface of a wall panel". N.B.R.I. Pretoria, S. Africa. 1950.

1.14. Roux, A.J.A., "Periodic heat flow through building components - Heat transfer from the outside surface of homogeneous wall panels to the inside air". N.B.R.I. Pretoria, S. Africa. 1950.

1.15. Roux, A.J.A., "Periodic heat flow through building components - Heat transfer from the outside surface of homogeneous wall panels to the inside air under winter conditions". N.B.R.I., Pretoria, S. Africa. 1950.

1.16. Roux, A.J.A., et al. "Periodic heat flow through building components - heat transfer through homogeneous wall panels from the Outdoor climate environment to the indoor air". N.B.R.I., Pretoria, S. Africa. 1951.

1.17. Holden, T.S., "Calculation of incident low temperature radiation upon building surfaces". A.S.H.R.A.E. Journal, Vol. 3, No.4, 1961.

1.18. Brunt, D., "Notes on radiation in the atmosphere". Quarterly Journal, Royal Meteorological Society, Vol. 58, 1932.

1.19. Swinbank, W.C., "Long-wave radiation from clear skies". C.S.I.R.O., Division of meteorological physics, Aspendale, Australia, 1963.

1.20. Hoglund, B.I., Mitalas, G.P., and Stephenson, D.G. "Surface temperatures and heat fluxes for flat roofs". Building Science, Pergamon Press, March, 1967.

1.21. Griffith, E., and Davis, A.H., "Transmission of heat by radiation and convection". Food Investigation Board, Special report No. 9, H.M.S.O. London, 1922.

1.22. Min, T.C., Schutrum, L.F., Parmelee, G.V., and Vouris, J.D., "Natural convection and radiation in a panel-heated room". Heating, Piping and Air Conditioning Journal, May, 1956.

1.23. Billington, N.S. "Building physics - Heat". Pergamon Press, London, 1967.

1.24. Parmelee, G.V., and Huebscher, R.G., "Forced convection heat transfer from flat surfaces". A.S.H.V.E. Research Bulletin, Vol. 53, No. 3, 1947.

1.25. Brown, A.I., and Marco, S.M. "Introduction to heat transfer" McGraw-Hill Book Co., New York, 1951.

1.26. Hottel, H.C. "Radiant heat transmission". Mechanical Engineering, Vol. 52, No. 7, July, 1930.

1.27. Oppenheimer, A.K., "Radiation analysis by the network method". A.S.M.E. Transactions, Vol. 78, 1956.

1.28. van Straaten, J.F., "Thermal performance of buildings". Architectural Science Series, Elsevier Publishing Co., London, 1967.

1.29. Rowely, F.B., Jordan, R.C., Lund, C.E., and Lander, R.M., "Gas is an important factor in the thermal conductivity of most insulating materials". A.S.H.V.E. Transactions, Vol. 58, 1952.

1.30. Lander, R.M., "Gas is an important factor in the thermal conductivity of most insulating materials - Part III", A.S.H.V.E. Transactions, Vol. 61, 1955.

1.31. Pratt, A.W., "Some observations on the variation of the thermal conductivity of porous inorganic solids with moisture content". Research series 30, current papers, B.R.S., D.S.I.R., 1964.

CHAPTER 2.

2.1. Mackey, C.O. and Wright, L.T. "Periodic heat flow: Composite walls and roofs". Heating, Piping and Air Conditioning, Vol. 18, No. 6, 1946.

2.2. Muncey, R.W. "The calculation of temperatures inside buildings having variable external conditions". Aust. J. Applied Science, Vol. 4, No. 2, 1953.

2.3. Pratt, A.W. and Ball, E.F., "Transient cooling of a heated enclosure". Int. J. Heat Mass Transfer, Vol. 6, 1963.

2.4. Raychaudhuri, B.C., "Transient thermal response of enclosures: The integrated thermal time-constant". International J. Heat Mass Transfer, Vol. 8, 1965.

2.5. Page, J.K., "Environmental research using models". Architect's J. Information Library, 11 March 1964.

2.6. Leopold, C.S., "Hydraulic analogue for the solution of problems of thermal storage, radiation, convection and conduction". A.S.H.V.E. Trans. Vol. 54., 1948.

2.7. Mackey, C.O. and Gay, N.R., "Cooling load from sunlit glass". A.S.H.V.E. Trans. Vol. 58, 1952.

2.8. Paschkis, Victor, "Periodic heat flow in Building walls determined by electrical analogy method" Heating, piping and air conditioning, Feb. 1942.

2.9. Buchberg, H., "Electric analogue prediction of the thermal behaviour of an inhabitable enclosure". A.S.H.R.A.E. Trans. Vol. 61, 1955.

2.10. Nottage, H.B., and Parmelee, G.V. "Circuit analysis applied to load estimating". A.S.H.R.A.E. Trans. Vol. 60, 1954.

2.11. Buchberg, H., "Electric analogue studies of single walls". A.S.H.R.A.E. Trans. Vol. 62, 1956.

2.12. Buchberg, H., "Circuit analysis applied to solar house-heating". Transactions of the Conference on the use of Solar Energy, Tucson, Arizona, U.S.A., Nov. 1955 - Standard Research Institute.

2.13. Thompson, K., and Hitchin, E.R., "Initial paper study of methods and equipment for an investigation of the thermal response of buildings to climatic variations. Unpublished report, B.C.R.U., Department of Building Science, Liverpool University, July, 1965.

2.14. Thompson, K., "Revised Simulator design". Unpublished report, B.C.R.U., Department of Building Science, Liverpool University, Nov., 1965.

2.15. Thompson, K. "A very high input impedence buffer using field effect transistors". Electronic Engineering, June, 1966.

CHAPTER 3.

3.1. Trewartha, G.T., "An introduction to climate". McGraw-Hill, New York, 1954.

3.2. Thornthwaite, C.W., "The climates of earth". Geog. Review, July, 1933.

3.3. Thornthwaite, C.W., "An approach toward a rational classification of climate". Geog. Review, Jan., 1948.

3.4. Bhalotra, Y.P.R. "Meteorology of Sudan". Memoir No. 6 Sudan Meteorological Service, Khartoum, 1963.

3.5. Sudan Meteorological Service, "Climate of the Sudan" Khartoum. Undated Note.

3.6. Roux, A.J.A., "A method of expressing climatic data and the use of these data in the design of buildings". Series D.R.093. N.B.R.I. Pretoria.

3.7. Richards, S.J. "Climatic control by building design". S.A. Architectural record, Vol. 44, No. 1, Jan., 1959.

3.8. Jordan, R.C., and Threlkeld, J.L., "Availability and utilization of Solar energy". Report No. 1502, A.S.H.V.E. Transactions, Vol. 60, 1954.

3.9. Moon, Parry, "Proposed standard radiation curves for engineering use". Journal of Franklin Institute, Vol. 230, Nov. 1940.

3.10. Rao, K.R., and Seshadri, T.N., "Solar insolation curves". Indian Journal of Meteorology and Geophysics, Vol. 12, No. 2, April, 1961.

3.11. Threlkeld, J.L., and Jordan, R.C., "Solar radiation during cloudless days". Report No. 1533, A.S.H.V.E. Transactions, Vol. 61, 1955.

3.12. Threlkeld, J.L., "Thermal environmental engineering". Prentice-Hall International, Englewood, N.J., 1962.

3.13. Hopes, "Solar charts for daylight planning" H.M.S.O. London, 1961.

3.14. Parmelee, G.V., "Irradiation of vertical and horizontal surfaces by diffuse solar radiation from cloudless skies". Report No. 1510, A.S.H.V.E. Transactions, Vol. 60, 1954.

3.15. Buchberg, H., and Naruishi, J., "Low income housing demonstration thermal analysis", Report 65-11, U.C.L.A., March, 1965.

3.16. MacPherson, R.K., "Physiological aspects of thermal comfort". Architectural Science Review, Vol. 8, No. 4, 1965.

3.17. Givoni, B., and Kim, Y., "Effect of the thermal environment and psychological factors upon subjects' response and performance of mental work". Ergonomics, Vol. 5, No. 1, 1962.

3.18. Bedford, T., "Basic principles of ventilation and heating". Lewis and Co., London, 1966.

3.19. Givoni, B., "Estimation of the effect of climate on man - Development of a new thermal index". B.R.S. Haifa, 1963.

3.20. Koch, W., "Humidity sensations in the thermal comfort range". Architectural Science Review, Vol. 6, No. 1., 1963.

3.21. Drysdale, J.W., "Climate and the design of buildings". Commonwealth Experimental Building Station, Sydney, Technical study No. 35, 1950.

3.22. MacPherson, R.K., "Studies in the preferred thermal environment" Architectural Science Review, Vol. 6, No. 4, 1963.

3.23. van Straaten, J.F., and van Deventer, E.N., "The functional aspects of building design in warm climates with particular reference to thermal and ventilation considerations". International Journal of Biometeorology, Vol. 8, No. 2, 1964.

3.24. Webb, C.G., "Thermal discomfort in a tropical environment". Research Series 22, B.R.S., London.

CHAPTER 4.

4.1. ASHRAE, "Guide and Data Book : Fundamentals and Equipment". 1963.

4.2. IHVE, "A guide to current practice", London, 1959.

4.3. Barned, J.R., "Thermal conductivities of building materials". Report No. 2, Division of building research, C.S.I.R.O., Melbourne, 1954.

4.4. Ingersoll, L.R., Zobel, O.J., and Ingersoll, A.C., "Heat conduction". Thames and Hudson, London, 1955.

4.5. Rowely, F.B., and Algren, A.B., "Thermal resistance of air spaces". ASHVE Transactions, Vol. 35, 1929.

4.6. Holden, T.S., and Greenland, J.J., "The coefficients of solar absorptivity and low temperature emissivity of various materials". Report No. 6, Division of Building Research, C.S.I.R.O., Melbourne, 1951.

4.7. Parmelee, G.V., "Application of meteorological data to indoor climate in buildings". Bulletin of the American Meteorological Society, No.6, Vol. 36, 1955.

4.8. Nottage, H.B., and Parmelee, G.V., "Circuit analysis applied to load estimating - Part II - Influence of transmitted solar radiation". Report No. 1529, ASHRAE Transactions, Vol. 61, 1955.

4.9. Saad, Shawki, and Hendry, A.W., "Measurements of soil temperatures and moisture contents". Civil engineering and public works review, Vol. 52, No. 609, March, 1957.

CHAPTER 5.

5.1. Raychaudhuri, B.C., Jain, S.P., and Yadva, K.G., "Thermal characteristics of unconditioned insulated masonary buildings in hot arid regions. Int. Journal of Biometeorology, Vol. 8, No. 2, 1964.

5.2. Givoni, B., and Shalon, R., "Influence of roof type and construction on indoor thermal conditions in Beeer Shiva". Research paper No. 11. Building Research Station, Haifa, 1962.

5.3. Roux, A.J.A., "Heat interchange between a roof and its surroundings". N.B.R.I. Bulletin No. 1, C.S.I.R., Pretoria, 1948.

5.4. van Straaten, J.F., "The thermal performance characteristics of curtain wall construction in warm climated". R.D. 33. C.S.I.R., Pretoria, 1961.

5.5. Raychaudhuri, B.C., Ali, S., and Garg, D.P., "Indoor climate of residential buildings in hot arid regions". Building Science, Vol. 1 Pergamon Press London, 1964.

5.6. Hendry, O.W., and Page, J.K., "Thermal movements and stresses in concrete slabs in relation to tropical conditions". B.R.S., Institute of Technology, Haifa, July, 1960.

5.7. Gupta, M.L., Gupta, C.L., Jain, S.P., and Raychaudhuri, B.C., "Periodic heat flow in conditioned structures". Indian Journal of Technology, Vol. 3, October, 1965.

5.8. Raychaudhuri, B.C., Jain, S.P., Gupta, C.L. and Gupta, M.L., "Heat transmission in insulated masonary structures under unsteady heat flow conditions". Indian Journal of Technology, Vol. 4, March, 1966.